5TH INTERNATIONAL MEETING OF THE APMP

ZURICH, SWITZERLAND | JANUARY 18 – 21, 2020

PROGRAM AND ABSTRACTS

Association for the Philosophy of Mathematical Practice

ETH zürich

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Jeremy Avigad | Carnegie Mellon University, USA
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Vincenzo De Risi | Laboratoire SPHère & CNRS, France
Gisele Secco | Universidad Federal de Santa Maria, Brazil

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CONFERENCE INFORMATION

Conference venue
The conference venue is the main building HG = Hauptgebäude of ETH Zürich. Main entrance: Rämistrasse 101.

The last session on Saturday will take place in the adjacent ML building. Main entrance: Sonneggstrasse 3.

Registration
Payment of registration fees covers participation in the meeting, congress materials, coffee break refreshments and buffet lunches.

Registration and Information desk
You can pick up your name badge and conference materials at the registration and information desk on the E floor of the main building.

The registration and information desk will be open at the following times:

Saturday: 08:15 –17:00
Sunday: 08:30-13:30
Monday: 08:30-17:00,
Tuesday: Contact organizers
Sara Booz: + 41 79 549 65 73

Instructions for speakers
- Duration of contributed talks: 40 minutes (including discussion)
- Duration of plenary lectures: 90 minutes (including discussion)

All conference rooms are equipped with PCs and projectors. You may upload your presentation to the conference computer from your USB flash drive or use your own computer (only VGA or HDMI plugs are available – mini HDMI and other connectors require an adapter).

Please use a standard format of presentation (ppt, pptx, or pdf). Please check that your computer or presentation work as expected during the break before your session.

Instructions for chairs
Please contact the speakers in advance so you can present them correctly and coordinate your timing signals.

Make sure to adhere to the advertised schedule so people can move between the parallel sessions.

Internet
You may connect to the *eduroam* network with your own institute's username and password. Please make sure to obtain the username and password in advance, as they may be different from your usual institutional username and password.

Registration on the networks *public* and *public-5* is possible for people who can receive text messages on their phones.

Tours, Sunday, January 19, afternoon
- Dada Tour: Departing at 15:30 from Cabaret Voltaire, Spiegelgasse 1 (old town).
- History and Money Tour: Departing at 15:00 from tram stop *Central*

Late registration at the information desk (20 CHF) will depend on availability. The registration desk will only accept cash in Swiss Francs.

Conference dinner Monday, January 20
The dinner will take place at Zunfthaus zur Schmieden, Marktgasse 20, starting at 20:00.

If you did not register for the dinner and would like to attend, please contact us as early as possible. Late registration at the information desk will depend on availability. The registration desk will only accept cash in Swiss Francs.

PRACTICAL INFORMATION

ATMs (cash machines)

The nearest ATMs are located in the MM building (adjacent to the conference venue, at the Polyterrasse) and in Rämistrasse 100 (across the road from the conference venue in the university hospital).

Emergency number

If you need the police or an ambulance when not at the venue, the emergency number is *112*.

Weather

Average daily high and low temperatures in January are 2 and -2 degrees Celsius respectively. Last year's high and low in January were 7 and -7. On average, there are 10 days of rain or snow in January.

TRAVEL INFORMATION

From Zurich Airport

- By tram to the conference venue: from the Zurich Airport tram stop, by tram no. 10 (towards Bahnhofplatz / HB) to ETH / Universitätsspital. The tram runs every 7 to 15 minutes between 6 o'clock in the morning and 11 o'clock at night. Journey time: 30 minutes.
- By rail to the Zurich HB (main station): by S-Bahn or mainline services from the Zurich Airport station. Journey time: approx. 10 minutes.

From Zurich HB (Main Station)

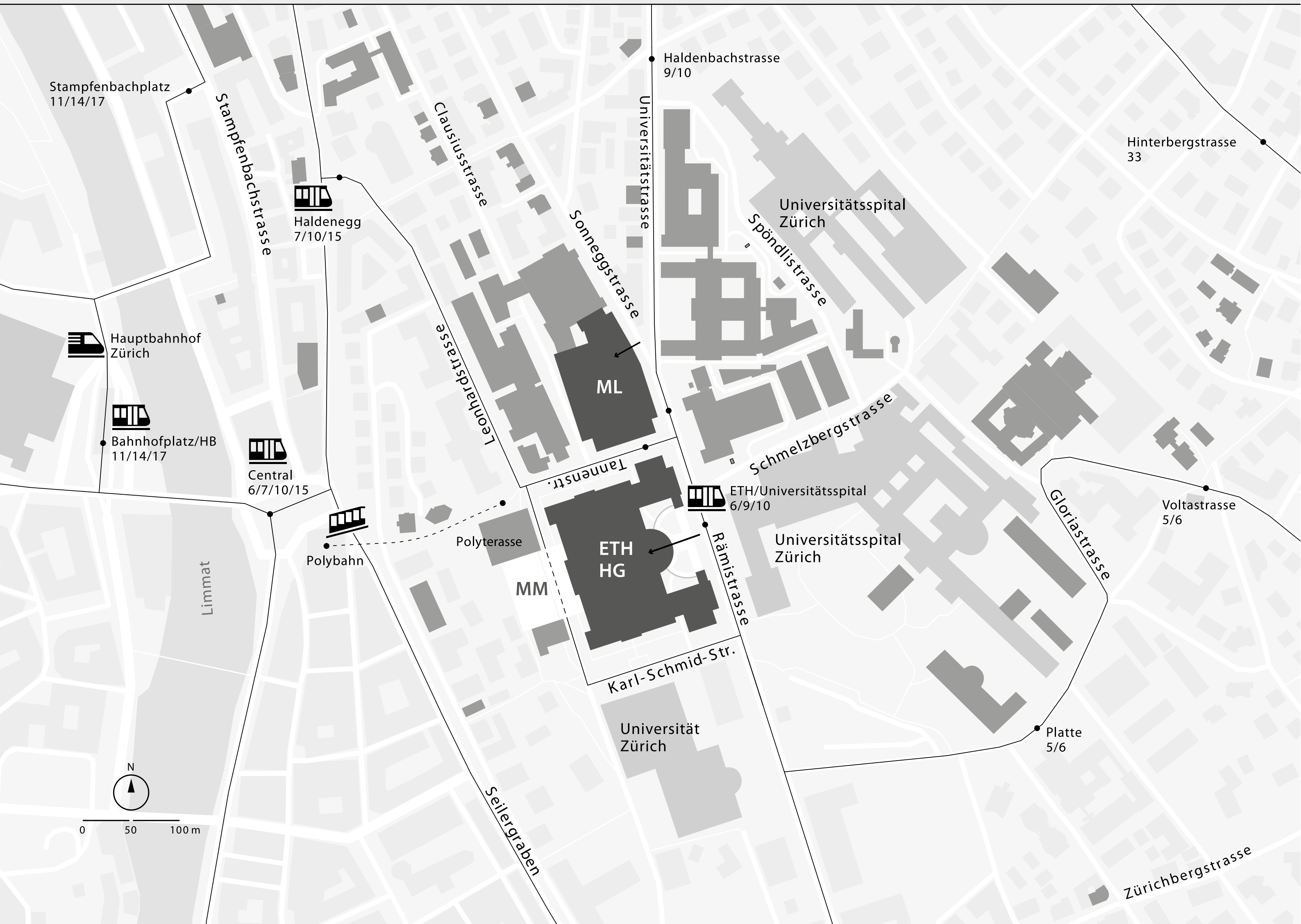
- From Bahnhofquai / HB tram stop: by tram no. 6 (direction: Zoo) to ETH / Universitätsspital. Journey time: approx. 6 minutes.
- From Bahnhofstrasse / HB tram stop: by tram no. 10 (direction: Airport or Oerlikon station) to ETH / Universitätsspital.
- Walking from the main station to the ETH main building takes ca. 13 minutes. Please note that the walk includes a rather steep uphill climb with some stairs, so you might want to avoid it if you're carrying luggage.

From Basel Airport / Basel Main Station (Bahnhof SBB)

- From Basel EuroAirport bus stop: bus no. 50 to Basel, Bahnhof SBB. There are busses every few minutes.
- At Basel, Bahnhof SBB take the IC3, ICE, or TGV to Zurich HB. Journey time: approx. 1 hour 20 minutes.

Zurich public transportation

- All travel within the city of Zurich is covered by the Zurich city ticket (zone 110).
- Travel from Zurich airport requires a ticket that covers zones 121 and 110.
- A single ride (zone 110) is valid for one hour and costs 4.30 CHF.
- A 24 hour ticket costs the same as two single rides.
- Tickets can be bought from machines in all public transport stations or from ticket-counters in the major stations.
- Public transportation in Zurich is usually punctual and dependable.



OPENING AND PLENARY SESSION | 08:45 – 10:30 | → HG E 5
Chair: Roy WAGNER | ETH Zürich
Opening statement

09:00 | → Page 41
Gisele SECCO | Universidade Federal de Santa Maria
Diagrams and computers in the proof of the Four-Color Theorem

COFFEE BREAK

PARALLEL SESSION 1A | 11:00 – 13:00 | → HG E 33.3
Chair: Gisele SECCO | Universidade Federal de Santa Maria

11:00 | → Page 48
Benjamin WILCK | Humboldt-Universität zu Berlin
Euclid's Philosophical Commitments

11:40 | → Page 18
Eduardo N. GIOVANNINI | Universidad Nacional del Litoral and Conicet
On the Cartesian significance of David Hilbert's Grundlagen der Geometrie

12:20 | → Page 17
Michele GINAMMI | University of Amsterdam
The interplay between physics and mathematics: from Dirac's delta to distribution theory

PARALLEL SESSION 1B | 11:00 – 13:00 | → HG E 33.1
Chair: Marianna ANTONUTTI MARFORI | Ludwig-Maximilians-Universität München

11:00 | → Page 05
Jessica CARTER | University of Southern Denmark
Fruitful representations in mathematical practice

11:40 | → Page 16
Valeria GIARDINO | CNRS
Representations and their cognitive significance in mathematics

12:20 | → Page 29
Danielle MACBETH | Haverford College
Diagrams and Figures in Ancient Mathematics: China and the West

LUNCH BREAK

PARALLEL SESSION 2A | 14:30 – 16:30 | → HG E 33.3
Chair: Mikkel Willum JOHANSEN | University of Copenhagen

14:20 | → Page 07
Murtaza CHOPRA | The Hebrew University of Jerusalem
"What is required should be done": some notes on cuneiform theory of mathematical practice.

15:00 | → Page 03
Viktor BLASJO | Utrecht University
Why did Greek geometers construct?

15:40 | → Page 08
João CORTESE & Taimara PASSERO | Universidade de São Paulo
On the importance of sensible matter for geometry: mathematical entities and procedures in Archimedes' heuristics

PARALLEL SESSION 2B | 14:30 – 16:30 | → HG E 33.1
Chair: Laura CROSILLA | University of Oslo

14:20 | → Page 12
Michael FRIEDMAN | Humboldt-Universität zu Berlin
How to notate a crossing of a braid? Notation as epistemic and / or as a hindrance

15:00 | → Page 34
Julien OUELLETTE-MICHAUD | McGill University
Notational bearings on conceptions of proofs

15:40 | → Page 46
David WASZEK | McGill University
"Informational equivalence" but "computational differences" of representations in mathematical practice

COFFEE BREAK

CHANGE BUILDING

PARALLEL SESSION 3A | 17:00 – 18:20 | → MLF 39
Chair: Robert THOMAS | University of Manitoba

17:00 | → Page 36
José Antonio PÉREZ ESCOBAR | ETH Zürich
Mathematical modelling and teleology in biology

17:40 | → Page 19
Emily GROSHOLZ | Pennsylvania State University
Mathematical Practice in Contemporary Biology: Field, Lab, Voting Booth

PARALLEL SESSION 3B | 17:00 – 18:20 | MLF → 38
Chair: Jemma LORENAT | Pitzer College

17:00 | → Page 10
Marlena FILA & Piotr BŁASZCZYK | Pedagogical University of Cracow
Limits of diagrammatic reasoning

17:40 | → Page 41
Michał SOCHAŃSKI | Adam Mickiewicz University
Visual computer experiments in mathematics—interpretations and philosophical issues

PLENARY SESSION | 09:00 – 10:30 | → HG E 5
Chair: Jessica CARTER | University of Southern Denmark

09:00 | → Page 28
Øystein LINNEBO | University of Oslo
Pluralities and sets in mathematical practice

COFFEE BREAK

PARALLEL SESSION 4A | 11:00 – 13:00 | → HG E 33.3
Chair: Øystein LINNEBO | University of Oslo

11:00 | → Page 06
Idit CHIKUREL | Tel Aviv University
Influences of Greek Geometrical Analysis on Maimon's Notions of Invention and Analysis

11:40 | → Page 25
Cornelia KNIELING | Carnegie Mellon University
Aristotle, Bolzano and the Question of Pure Proofs

12:20 | → Page 37
Tabea ROHR | University of Jena
Geometrical Practice between Unification and Purity of Methods. A 19th century case study

PARALLEL SESSION 4B | 11:00 – 13:00 | → HG E 33.1
Chair: Paola CANTU | Aix-Marseille Universite and CNRS

11:00 | → Page 39
Dirk SCHLIMM | McGill University
What the study of notations can tell us about mathematical practice

11:40 | → Page 24
Anna KIEL STEENSEN | ETH Zürich
Textual proof practices in Dedekind's early theory of ideals

12:20 | → Page 27
Javier LEGRIS | University of Buenos Aires & CONICET
Charles S. Peirce on Identity: From algebra to diagrams

SANDWICH LUNCH

APMP PLENARY BUSINESS MEETING | 13:30 – 14:30 | → HG E 5

HISTORY AND MONEY TOUR
START: 15:00
TRAM STOP "CENTRAL"

DADA TOUR
START: 15:30
CABARET VOLTAIRE
SPIEGELGASSE 1

PLENARY SESSION | 09:00–10:30 | → HG E 5
Chair: Sascha FREYBERG | Università Ca' Foscari di Venezia & Max Planck Institute for the History of Science

09:00 | → Page 28
Jemma LORENAT | Pitzer College
Mathematics or moonshine: non-Euclidean geometry in The Monist at the beginning of the twentieth century

COFFEE BREAK

PARALLEL SESSION 5A | 11:00–13:00 | → HG E 33.3
Chair: Jose Ferreiros | Universidad de Sevilla

11:00 | → Page 26
Brendan LARVOR | University of Hertfordshire
David Hume and the Limits of Mathematical Reason

11:40 | → Page 30
Kenneth MANDERS | University of Pittsburgh
Mathematical "Error" in Descartes: Failure in Algorithmic-Exploratory Practice

12:20 | → Page 03
Sandra BELLA | Université Paris-Diderot, Laboratoire SPHERE
Making sense of the impossibility 0/0, ca. 1700

PARALLEL SESSION 5B | 11:00 - 13:00 | → HG E 33.1
Chair: Danielle MACBETH | Haverford College

11:00 | → Page 25
Ladislav KVASZ | Institute of Philosophy & Czech Academy of Sciences
How can abstract objects of mathematics be known?

11:40 | → Page 40
Pierrot SEBAN | Université Paris Nanterre
Two ways to mathematical objectivity: how to salvage a philosopher's insight?

12:20 | → Page 33
Matías OSTA-VÉLEZ | Ludwig-Maximilians-Universität
Guillermo NIGRO | Universidad de la República
Grounding mathematical concepts in practices: language games and conceptual development

LUNCH BREAK

PARALLEL SESSION 6A | 14:30–16:30 | → HG E 33.3
Chair: Matthias SCHEMMEL | Max Planck Institute for the History of Science

14:30 | → Page 31
Raziehsadat MOUSAVI | Max Planck Institute for the History of Science
Calendrical Reform and Functionalism: Engagement of Mathematical Astronomers in Executive Practices in Early Islamic Time

15:10 | → Page 32
Pietro Daniel OMODEO | Ca' Foscari University of Venice
Senthil Babu | French Institute of Pondicherry
A Copernican Revolution in the Lagoon: When A Galilean Mathematician Tried to Solve the Hydrogeological Problems of Venice

15:50 | → Page 43
Marco STORNI | Ca' Foscari University of Venice
The Map of France and the Shape of the Earth: the Eighteenth-Century Debate over Cartography, Mathematical Practices and Cosmology in the Paris Academy

PARALLEL SESSION 6B | 14:30–16:30 | → HG E 33.1
Chair: Dirk SCHLIMM | McGill University

14:30 | → Page 42
Henrik Kragh SØRENSEN & Mikkel Willum JOHANSEN | University of Copenhagen
The BMI and mathematical practice: Abel's exception, history of infinity and cognitive accounts of mathematics

15:10 | → Page 14
Manuel Jesús GARCÍA-PÉREZ & José FERREIRÓS | University of Sevilla
The emergence of geometric knowledge: an interdisciplinary approach

15:50 | → Page 35
Jean-Charles PELLAND | New College of the Humanities
Recipes for talking about mathematical progress

COFFEE BREAK

PARALLEL SESSION 7A | 17:00–18:20 | → HG E 33.3
Chair: Vincenzo De Risi | Université Paris-Diderot & Laboratoire SPHERE

17:00 | → Page 11
Sascha FREYBERG | Università Ca' Foscari di Venezia & Max Planck Institute for the History of Science
Matthias SCHEMMEL | Max Planck Institute for the History of Science
On the dialectics of abstraction as a cognitive and historical process

17:40 | → Page 13
Elías FUENTES GUILLÉN | UNAM & Czech Academy of Sciences
Davide CRIPPA & Jan MAKOVSKÝ | Czech Academy of Sciences
"Give me a lever": Bolzano and the practice of applied mathematics in Prague at the beginning of the 19th century

PARALLEL SESSION 7B | 17:00–18:20 | → HG E 33.1
Chair: Keith WEBER | Rutgers University

17:00 | → Page 44
Dragan TRNINIC & Manu KAPUR | ETH Zürich
Exploring the efficacy of practice before instruction

17:40 | → Page 21
Karl HEUER | Technical University of Berlin
Deniz SARIKAYA | University of Hamburg
How the tangible gets abstracts and vice versa: Experience from classes with mathematically gifted youth

CONFERENCE DINNER 20:00

PLENARY SESSION | 09:00 – 10:30 | HG E 5
Chair: Brendan LARVOR | University of Hertfordshire

09:00 | → Page 02
Jeremy AVIGAD | Carnegie Mellon University
Reliability of mathematical inference

COFFEE BREAK

PARALLEL SESSION 8A | 11:00 – 13:00 | → HG E 33.3
Chair: Juan Luis GASTALDI | ETH Zürich

11:00 | → Page 19
Henning HELLER | University of Vienna
From the group concept to group theory

11:40 | → Page 04
Paola CANTU | Aix-Marseille Université and CNRS
Frédéric PATRAS | Université Nice Sophia Antipolis & CNRS
Bourbaki's mathematical practice and the distinction between philosophical and mathematical structuralism

12:20 | → Page 45
Thomas TULINSKI | ENS de Lyon
On abstraction theorems for homotopy categories

PARALLEL SESSION 8B | 11:00 – 13:00 | → HG E 33.1
Chair: Henrik Kragh SØRENSEN | University of Copenhagen

11:00 | → Page 09
Silvia DE TOFFOLI | Princeton University
A Fallibilist Account of Mathematical Justification

11:40 | → Page 02
Zoe ASHTON | The Ohio State University
Developing Dots: The Role of Audience in Proof Methods

12:20 | → Page 38
Bernhard FISSENI | Institut für Deutsche Sprache
Deniz SARIKAYA | University of Hamburg
Bernhard SCHRÖDER | Universität Duisburg-Essen
Martin SCHMITT | Ludwig-Maximilians-Universität München
How to Frame a Mathematician: Modelling the Cognitive Background of Proofs

LUNCH BREAK

PARALLEL SESSION 9A | 14:30 – 16:30 | → HG E 33.3
Chair: Jeremy AVIGAD | Carnegie Mellon University

14:30 | → Page 22
Mikkel Willum JOHANSEN | University of Copenhagen
Henrik Kragh SØRENSEN | University of Copenhagen
ML to the rescue for PMP? Using machine learning in large-scale quantitative investigations of mathematical diagrams

15:10 | → Page 15
Juan Luis GASTALDI | ETH Zürich
Mathematical Language Processing: A conceptual and technical framework for the automatic treatment of mathematical texts

15:50 | → Page 23
Deborah KANT | University of Konstanz
How does a qualitative interview study inform the philosophy of set theory?

PARALLEL SESSION 9B | 14:30 – 16:30 | → HG E 33.1
Chair: Senthil Babu | French Institute of Pondicherry

14:30 | → Page 47
Keith WEBER | Rutgers University
Essentially informal proofs about infinite time Turing machines

15:10 | → Page 01
Marianna ANTONUTTI MARFORI | Ludwig-Maximilians-Universität München
Explanatoriness and the de re content of proofs

15:50 | → Page 08
Laura CROSILLA | University of Oslo
Generalized predicativity

COFFEE BREAK

PLENARY SESSION | 17:00 - 18:30 | → HG E 5
Chair: Valeria GIARDINO | CNRS

17:00 | → Page 09
Vincenzo DE RISI | Université Paris-Diderot & Laboratoire SPHERE
The theory and practice of space: interactions between epistemology and expertise in early modern geometry

Marianna ANTONUTTI MARFORI
Ludwig-Maximilians-Universität München | marianna.antonutti@gmail.com
Tuesday, January 21 | 15:10 – 15:50 | Parallel session 9b | → HG E 33.1
Explanatoriness and the de re content of proofs

In this talk, I will introduce a new approach that aims to track the "informational" content of mathematical arguments, and I will argue that this approach sheds light on certain debates in the philosophy of mathematical practice.
First, I will propose that most theoretical virtues of proof can be characterised either as agent-relative (depending on the agent's cognitive setup and preferences), non-agent-relative (depending on the mathematical features independent of the cognitive agent who devises or studies the proof), or a combination thereof. Examples include, respectively, surveyability (agent-relative), purity (non-agent-relative), and explanatoriness (where this can be said to be a "complex" theoretical virtue, i.e. one that can be explained, at leat to some extent, in terms of simpler theoretical virtues).
I propose to capture the "informational" content of mathematical proofs by applying the distinction between "de re" and "de dicto" predication to the analysis of mathematical statements. A theorem stating the existence of a mathematical object can provide a merely existential result, or it can carry additional information concerning the identity of the object that is proven to exist. For example, consider the theorem that T has an infinite path through it, where T is an infinite binary recursive tree. Depending on the proof available to us, we can obtain the knowledge that there is a set X such that X is a path through T (in which case the property of ‚being a path through T' is attributed to the set "de dicto", and we do not have access to other properties of X), or that there is a specific, defined set X such that X is a path through T (in which case the property is attributed to the set "de re", and X satisfies an explicit definition). In this sense, a proof will be said to provide "de dicto knowledge" of a mathematical statement if it provides knowledge of a purely existential statement, whereas a proof provides "de re knowledge" when it carries additional information concerning the identity of the object that is proven to exist.
Second, I will argue that providing de re or de dicto knowledge is a non-agent-relative property of proofs that can be used to explain complex theoretical virtues of proofs. In particular, I will suggest that the virtue of explanatoriness could be attributed to proofs that are (i) surveyable (i.e. they can be followed by a competent agent with finite cognitive powers), (ii) persuasive (i.e. they provide understanding of the conclusion relative to the agent cognitive setup), and (iii) provide de re knowledge of their conclusion. In concluding, I will argue that this approach has unificatory power in that it can be used to explain why certain kinds of reasoning—namely, those that provide "de re knowledge"—are particularly useful in mathematics independently of their modality (inferential, visual, programming language, etc.), and that it explains the common belief that certain computer proofs are not explanatory on the grounds that they provide understanding and de re knowledge of their conclusion, but they are not surveyable.

Zoe ASHTON
The Ohio State University | ashton.95@osu.edu
Tuesday, January 21 | 11:40 – 12:20 | Parallel session 8b | → HG E 33.1
Developing dots: the role of audience in proof methods

The role of audiences in mathematical proof has largely been neglected, in part due to misconceptions like those in Perelman & Olbrechts-Tyteca (1969) which bar mathematical proofs from bearing reflections of audience consideration. In this paper, I argue that mathematical proof is typically argumentation and that a mathematician develops a proof with an audience in mind. The specific audience she has in mind while proving is the universal audience. In so doing, he creates a proof which reflects the standards of reasonableness embodied in his universal audience. This universal audience is a rhetorical concept which derives from Perelman & Olbrechts-Tyteca's original formulation. Arguers construct their arguments to the universal audience when they aim to convince all people over time and place. But each arguer can only construct his concept of what is universal from his experiences with particular audiences. As a result, it is a concept which retains the goal of universality, something I argue mathematicians aim for, while allowing the realizations of the universal audience to be influenced by particular audiences. Overall I argue that this rhetorical concept is an accurate and useful way of understanding mathematical proof development.

Given this framework, we can better understand the introduction of proof methods based on the mathematician's likely universal audience. I examine a case study from Alexander and Briggs's work on knot invariants to show that we can fruitfully reconstruct mathematical methods in terms of audiences. I specifically focus on their use of a dotting notation in the construction of knot diagrams. They claim that such a method is superior to the broken line notation, one familiar to us today. I argue that these claims are best understood in terms of the particular audiences that would have heavily influenced his constructed universal audience.

Jeremy AVIGAD
Carnegie Mellon University | avigad@cmu.edu
Tuesday, January 21 | 09:00 – 10:30 | Plenary Session | → HG E 5
Reliability of mathematical inference

Of all the demands that mathematics imposes on its practitioners, one of the most fundamental is that proofs ought to be correct. This is also a demand that is especially hard to fulfill, given the fragility and complexity of mathematical proof. This essay considers some of ways that mathematics supports reliable assessment, which is necessary to maintain the coherence and stability of the practice.

Sandra BELLA
Université Paris-Diderot, Laboratoire SPHERE | bellusky@hotmail.com
Monday, January 20 | 12:20 – 13:00 | Parallel session 5a | → HG E 33.3
Making sense of the impossibility 0/0, ca. 1700

Shortly after being introduced to the Leibnizian calculus in December 1691, Guillaume de l'Hospital (1661 – 1704) faces a quotient whose numerator and denominator become both equal to zero. His mathematical mentor, Johann Bernoulli (1667 – 1748), shows him how the differential calculus raises the indeterminacy by taking the quotient of the differential of the numerator by that of the denominator. This precious rule is enshrined in Article 163 of the well-known treatise Analyse des infiniment petits pour l'intelligence des lignes courbes, published by l'Hospital in 1696.
In July 1700 Michel Rolle (1652 – 1719) attacks the Leibnizian Calculus before the Académie royale des Sciences. He questions both the "fundamental assumptions of the geometry of the infinitely small" and the "exactness" of the differential calculus. Rolle puts the new calculus to the test by providing a number of examples of curves on which the determination of extrema or tangents by the differential calculus leads, according to him, to insufficient results—results which, moreover, call into question the new calculus's exactness. However, such cases provide the opportunity for the differential calculus to show its advantages, especially in the determination of tangents at a point of "décussation" (double cusp point) in which the ratio 0/0 appears thus interpreted geometrically.
If in a purely algebraic framework the ratio 0/0 makes no sense—because it has no meaning—, the differential calculus provides a new framework in which it is possible to interpret the quotient geometrically, thereby legitimizing its computational processing. We will show that this construction of meaning becomes a key argument that highlights the specifity of the new calculus, an argument which, as we shall see, Leibniz will develop in its exchanges, private and public.

Viktor BLASJO
Utrecht University | v.n.e.blasjo@uu.nl
Saturday, January 18 | 15:00 15:40 | Parallel session 2a | → HG E 33.3
Why did Greek geometers construct?

Why did Greek mathematicians think it was a good idea to spend hundreds of years trying to make an angle the third of another, or a cube twice the volume of another, in dozens of different ways? What sin could be so grave that they imposed on themselves such a Sisyphean task? Why make things at all, and why do so only sometimes, with Janus-faced inconsistency? Why meticulously articulate recipes for transferring line segments by ruler and compass, only to then suddenly move entire triangles like it's nobody's business in the very next proposition, as Euclid seemingly does in Elements I.4? And how can Archytas consider the cube root of 2 a mystery wrapped in an enigma, but then at the same time think that taking the intersection of a torus and a cylinder is a piece of cake, as he ostensibly does in his cube duplication?

I argue that core foundational concerns motivated this pursuit. Constructions ensure consistency, validate diagrammatic reasoning, protect against hidden assumptions, and, most radically, suggest an all-out operationalist theory of meaning of mathematical concepts. The latter in particular enables us to push a constructivist reading of Greek mathematics further than has been done previously. For instance, it enables us to interpret Euclid's use of superposition as actual reconstruction, which has long been thought unsustainable.

I analysed the complete corpus of solutions to the three classical construction problems from the point of view of these foundational purposes of constructions. I argue that technical aspects of these solutions strongly hint at such philosophical motivations, even though that is never explicit in surviving sources. This suggests new interpretations that reconstruct operationalist aspects of the solutions that were ignored or not understood by the commentators who preserved them.

For example, the account of Archytas's cube duplication that has come down to us is perplexing, not to say conceptually incoherent, since it appears to be based on taking for granted things that are vastly more complicated than the problem itself. Archytas "constructs" the cube root of 2 only on the assumption that very complicated intersections of various surfaces can be taken at will. The intersections, it would seem from the text, are assumed to become immediately available to us merely by being defined. If one can call into being by simple decree such a complicated object as the curve of intersection of a cylinder and a torus, then why can one not do the same with a segment of a certain length or a cube of a certain volume?

In fact, Archytas's solution can be interpreted as a mechanism that produces the solution by rulers pushing one another until they deterministically lock into the solution configuration. This is even a "one degree of freedom" or "single-motion" mechanism: an important condition for foundational purposes, as later emphasised by Descartes and others.

Similar operationalist reinterpretations suggest themselves for other Greek solutions of the classical problems, thereby strengthening the case for my reconstruction of the underlying philosophy.

Paola CANTU & Frédéric PATRAS
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Aix-Marseille Université and CNRS | patras@unice.fr
Tuesday, January 21 | 11:40 – 12:20 | Parallel session 8a | → HG E 33.3
Bourbaki's mathematical practice and the distinction between philosophical and mathematical structuralism

There is a widespread tendency in the philosophy of mathematics to distinguish between a methodological (often also called mathematical) and a philosophical structuralism [Reck and Price, 2000], but the distinction is not yet generally accepted nor

always coherent, especially because it is unclear whether it should be based on different notions of structure or on different uses of the same notion. Methodological structuralism is generally associated with an analysis of the method that is applied by mathematicians when they are doing mathematics and that has evolved in time (e.g. the use of informal or axiomatic presentations, the role of intuition and formal deductions, the relation between alternative ways to frame mathematics using set theory or category theory). Philosophical structuralism is used as a collective name for a large number of different philosophical theories centering on the fundamental question: "What is a structure?", and investigating issues such as the difference between objects and structures, or what it means for an axiomatically described structure to be 'formal'. So, the two approaches are not clearly distinguishable, because methodological structuralism tackles deep philosophical questions, whereas philosophical structuralism discusses methodological issues. Given that the notion of structure, or, as we will see, the notions of structure (in the plural) cannot be disentangled from their origin in specific theories and epistemological approaches developed by mathematicians themselves, the philosophy of mathematical practice and historically informed analysis of specific case studies might be particularly insightful to address the issue of structuralism (see e.g. Carter [2008]).
Bourbaki's work is particularly interesting from this point of view. His views and ideas on structures are often mentioned as a relevant tradition and source of structuralism (see for example Isaacson [2008] and Shapiro [1997]) but rarely discussed in detail (exception made for the quite old paper by Corry [1992]). The group always advocated that what he "considered as important is communication between mathematicians, personal philosophical conceptions being irrelevant for him" [Dieudonné, 1982, p. 618]. We will show that in many respects one can argue that Bourbaki consistently claimed that his conceptions and in particular his structuralist views were driven by mathematical practice, as for example when he remarked that the Éléments de mathématique aimed at furnishing "a bag of tools, a tool kit for the working mathematician" [Dieudonné, 1982, p. 620]. The analysis of this claim, deeply rooted in Bourbaki's actual daily work, and in the experience of collective writing of a mathematical encyclopedic treatise that would have made him a "new Euclid", together with the `architectural' role assigned to structures, will be shown to have manifold implications for the philosophy of mathematical practice, for our understanding of (mathematical and philosophical) structuralism, and for their interplay.

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Saturday, January 18 | 11:00 – 11:40 | Parallel session 1b | → HG E 33.1
Fruitful representations in mathematical practice

I will address the question: What are the properties of (fruitful) representations used in mathematical practice? It has been claimed that graphical representations offer "free rides" (Atsushi Shimojima) and that new figures "pop up" as the result of constructions made in Euclidean diagrams (Ken Manders). This indicates that graphical, or diagrammatic, representations are fruitful representations in contrast to linguistic or sentential

representations. But recently Danielle Macbeth has argued that new "objects" can also "pop up" when manipulating mathematical expressions. Another observation is that mathematical notations, formulae and expressions also possess certain visual features, so that they in some ways resemble diagrams. The main question to be addressed is how to characterise these representations, that is, mathematical expressions and formulae. It will first be noted that they may function in quite distinct ways. They may, for example, mainly symbolically, record information. Or the notation could be an iconic representation of the very objects of reasoning, as the arrows in category theory. In order to address this question, I will propose a framework consisting of placing various representations in a coordinate system. The first axis notes the type of representation, lying between a linguistic, sentential representation and a graphic representation. The second axis measures the representation according to the level of iconicity, ending in a symbolic representation. The usefulness of this framework in terms of determining properties of fruitful representations will be discussed. And, of course, a number of different examples of representations used in mathematical practice will be given.

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Sunday, January | 11:00 – 11:40 | Parallel session 4a | → HG E 33.3
Influences of Greek Geometrical Analysis on Maimon's Notions of Invention and Analysis

In 1795, Salomon Maimon published two articles describing the outlines for his theory of invention. He intended to publish the complete work titled Perfection of the Inventive Faculty through the Study of Mathematics, but unfortunately never did. My work presents a reconstruction of this theory. A significant part of Maimon's theory of invention is concerned with presenting methods of invention to be used in mathematics. The majority of these methods are methods of analysis whereas there are only few methods of synthesis. Maimon turned to Euclidean geometry and practices of Greek geometrical analysis as his main source of influence. More specifically, he was influenced by Proclus' commentary on Book I of Elements and by practices of diorism.
This influence is extended not only to methods but also to his notions of invention and analysis. According to Maimon, even though invention is grounded on logical analysis, we are often required to use other kinds of analysis in order to arrive at new mathematical proofs and solutions. Thus he presents two notions of analysis: one grounded on the principle of contradiction alone and one grounded on intuition as well. The formation of the latter is the result of a direct influence of mathematical practices of analysis. Therefore, my discussion is accompanied by examples taken from Euclid's Elements and Data.

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Saturday, January 18 | 14:20 – 15:00 | Parallel session 2a | → HG E 33.3
"What is required should be done": some notes on cuneiform theory of mathematical practice

Through studying mathematical and astronomical texts of the cuneiform traditions that are written on clay tablets, I was surprised by how much clearly they make manifest the "magical" aspect of mathematical reasoning. This inappropriate expression tries to point to the use of unexpected, seemingly too simple relations between mathematical objects. This tradition yields another perspective on very classical epistemological questions, because of its being always interested in doing and not in establishing a propositional background. In this paper, I want to focus on three of these questions. The first one is, what is the importance of logical foundation for mathematics? The second one is, do we discover or do we create mathematical objects? The third would be: what relation is there between mathematics and our experiencing of the world? I will deal with the first illustrating example in details and will present the others according to the time I'll have.

I will use an example of undetermined analysis that appear in an old babylonian tablet and that was already studied by Hoyrup. He already pin-pointed the virtuoso nature of this text, though after all it deals with a quite simple question, if we formulate it in our algebra. But, its taste will really appear if we follow it, as it is, as much as possible. I want to go further than Hoyrup in his interpretation. He has noticed that the result is easy to guess and so, the result is not the purpose of this text. I propose that the purpose of this texts is to express a theory that is not propositional but fundamentally consists in what we would call tricks. In particular, this interpretation explains the central function of ambiguities that our logical norms would struggle to exclude.
In a second part, I'll try to present the very convincing reconstruction of lunar system A, in mathematical astronomy that J. Britton has proposed. It is an occasion to show how much, in taking numbers as first polysemic objects, the constructed ad hoc character of solutions is apparent. One creates tools and solutions that will produce nice results. It is an occasion to ask whether we are mistaken when we think that geometric properties of objects that seem more to come from the objects themselves. What does it mean, for instance, that the complete determination of pythagorean triplets historically preceded the pure geometric demonstration of side of right triangle property?

Thirdly, the most surprising is that these very constructive methods do not sound artificial at all. I will try to show that this results from the context of their construction. They resound with other parts of scribal culture. In one way, a lot of "playing" possibilities are permitted in cuneiform practice but in a very normative frame. These norms give its sense to mathematical practice and are effectively experienced by it.
In conclusions, I want to raise the question: what makes this practice together so strange and so familiar for us?

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On the importance of sensible matter for geometry: mathematical entities and procedures in Archimedes' heuristics

Although Plato had used in Timaeus some quantitative, mechanical and mathematical models, he did it mainly as an aspect of his finalist and qualitative cosmology. Plato did not develop in a deep way some quantitative conception of matter and an application of mathematical analysis to physics. Traditionally, Greek mathematical analysis (which is always followed by a synthesis) is divided in analysis of problems and analysis of theorems. The first concerns to the search of the exact way to build a given figure, and the second is related to the search of a theorem demonstration that is already formulated. However, according to Martin (1992, p. 171), there is a third case of analysis, the most important one, that is the looking for the proposition of the theorem itself which will be later demonstrated. This third type of analysis is represented by Archimedes' Method. Archimedes writes to Eratosthenes that by means of his Method "you will be enabled to recognize certain mathematical questions with the aid of mechanics" (apud Dijksterhuis 1987, p. 314). Notably, Archimedes uses the lever law in the determination of centers of gravity - that is to say, of geometrical figures! What does it mean to make this appropriation of mechanics in the realm of geometry? Although Archimedes did not made demonstrations using a mechanical approach, his heuristics on The Method were based on it (Detlefsen, 2008). Should it be considered to have a merely heuristic role or should it enter on the construction of the geometrical domain itself? Taking into account that Archimedes connected the mathematical analysis with the sensory experience, our purpose in this communication is to discuss the way that Archimedes dealt with mathematical entities; the relation between Archimedes' geometrical vocabulary and the domain of vision; and finally, the role of drawing diagrams in Archimedes mathematical heuristics. We base ourselves both on Archimedes' argumentation presented on his Method and on some discussion about the connection between sensory experience and geometrical purity in the vocabulary of the Stomachion.

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Generalized predicativity

Constructive theories of sets such as Aczel and Myhill constructive set theory and Martin-Löf type theory are said to be generalised predicative. I discuss the notion of generalised predicativity and compare it with the classical notion of predicativity that was analysed by Kreisel, Feferman and Schütte starting from the 1950's. A significant difference between the classical and the constructive notions of predicativity, is that so-called generalised inductive definitions are considered (in general) impredicative from a classical prespective, but are accepted as predicative from a constructive prespective. I will propose an analysis of this fact that employs concepts from the early historical debates on predicativity, and especially the reflection by the late Poincaré. I will suggest that Poincaré's thought offers a characterisation of predicativity that better suits contemporary manifestations of generalised predicativity, compared with the well-known characterisation of this notion in terms of lack of vicious circles.

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Tuesday, January 21 | 17:00 – 18:30 | Plenary Session | → HG E 5
The theory and practice of space: interactions between epistemology and expertise in early modern geometry

The talk investigates the changing views on diagrams, axioms, and space in early modern elementary geometry. Different conceptions of space, mainly provided by metaphysical investigations, seem to have gradually changed the meaning of axioms in the epistemology of mathematics, while the latter transformed the role played by diagrams in actual geometrical demonstrations. On the other hand, a system of well-established practices already regulated the use of diagrams and fixed the standards of rigor in early modern geometry. We will explore how new epistemological ideas conflicted with mathematical practices, and how they eventually changed the latter by establishing new standards and tools. This should shed some light on the relations between mathematical practice and mathematical epistemology in the course of history.

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A Fallibilist Account of Mathematical Justification

In my talk, I will put forward an account of mathematical justification that is faithful to actual mathematical practice. I will focus on mathematical doxastic justification, that is, justification for an agent's belief in a mathematical claim for mathematical reasons. In contrast to traditional views, I will argue that even in the case of mathematics justification and knowledge can come apart and that therefore the doors are open to Gettier type of cases. I will argue that the norms for doxastic justification at play in actual mathematical practice apply to individual agents but present an important social component as well. Moreover, in my view the bar on justification changes according to the social role the agent is playing. Whereas for the laywoman pure testimony is enough and for the clairvoyant the reliability of her super-power would suffices, for the expert mathematician a mathematical argument is needed. Such argument is what I label a simil-proof (SP), that is, an argument that looks like a proof to the relevant agents. I will characterize SPs as sharable: having a SP implies grasping how it supports its conclusion and also being able to share it in the appropriate context. This implies that being justified is connected to the ability not only of responding to criticism adequately, but also of justifying. One striking respect in which my account of mathematical justification differs from more traditional ones

is that it has a fallibilist flavor: justification comes apart from truth since an agent may be justified in believing a false proposition or in believing a true proposition by improperly grasping a fallacious argument.

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Saturday, January 18 | 17:00 – 17:40 | Parallel session 3b | → ML F 38
Limits of diagrammatic reasoning

We challenge theses of (Brown, 1997) and (Giaquinto, 2011) concerning the Intermediate Value Theorem (IVT); we argue that a diagrammatic reasoning is reliable provided one finds a formula representing the diagram.

IVT states: If $(F,+,\cdot,0,1,<)$ is an ordered field, $f: [0,1] \to F$ is a continuous map such that $f(0) f(1) < 0$, then $f(x) = 0$, for some $x \in (0,1)$. An accompanying diagram, diag(IVT), depicts a graph of f intersecting a line $(F,<)$, as the function values differ in sign.

(a) In (Brown, 1997), Brown argues that diag(IVT) guarantees the existence of an intersection point. (b) In (Giaquinto, 2011), Giaquinto argues that diag(IVT) do not guarantee the existence thesis, since continuous functions include non-smooth functions that find no graphic representations. (c) Both Brown and Giaquinto believe that Bolzano sought to prove IVT.

(ad a) We show that IVT is equivalent to Dedekind Cuts principle (DC):
If (A,B) is a Dedekind cut in $(F,<)$, then $(\exists! c \in F)(\forall x \in A)(\forall y \in B)[x \leq c \leq y]$.

We also provide a graphic representation for DC. This equivalence justifies the claim that IVT is as obvious as DC. There is, however, no relation between diag(IVT) and diag(DC), all the more between diag(IVT) and the formula DC. Thus, Brown's claim has to be based on the analytic truth IVT $\leftrightarrow$ DC.

(ad b) Diagrams representing lines $(F,<)$ do not depict whether the field $(F,+,\cdot,0,1,<)$ is Euclidean (closed under the square root operation), or $(\mathbb{R},+,\cdot,0,1,<)$, or a real-closed field; graphs of f do not distinguish between polynomial and smooth functions. IVT for polynomials, IVTp, is valid in real-closed fields (these fields could be bigger or smaller than real numbers); in fact, IVTp is the axiom for real-closed fields (next to the Euclidean condition).

(ad c) We provide a logical structure of (Bolzano, 1817) and show that in fact, Bolzano sought to prove IVTp, whilst IVT was just the lemma. We address the question: Why did Bolzano sought to prove IVTp rather than a more general problem, namely IVT.

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Monday, January 20 | 17:00 – 17:40 | Parallel session 7a | → HG E 33.3
On the dialectics of abstraction as a cognitive and historical process

Studies on the emergence of the concept of number and its further historical development suggest that the earliest forms of 'supra-utilitarian' mathematics, i.e., institutionalized mathematics not immediately directed at a practical goal, developed in the context of administrative activities within the increasingly stratified societies of early civilizations, in particular in Mesopotamia. They resulted from the exploration of cognitive structures inherent in the symbolic and material means of state administration and became increasingly detached from their original practical contexts, eventually bringing about abstract concepts (such as that of number), with more general validity and more universal applicability. The history of mathematics in praxis thus appears to present us with a telos towards universality. However, this scant synopsis already gives rise to a couple of fundamental questions concerning mathematical practices in their epistemic, material and social dimensions. Indeed, objections have been raised concerning the claim of universality, countering its 'whiggish' perspective with an alternative take on abstraction as a process of exclusion, which implies a cognitive as well as a socio-political dimension.

Now, are these claims contradicting each other or are they referring to simultaneous processes of different levels of analysis? In the perspective of a long-term history of knowledge the problem of mediating genesis and validity, historical and logical aspects is not unusual. On the one hand we need to understand knowledge tools and their emergence within a horizon of possibilities and consider the path-dependency of historical developments, which always build upon existing material-symbolic systems and always take place within a specific societal-institutional setting. Notwithstanding the contingency of historical developments, which are based on specific societal conditions, abstraction processes obviously do present us on the other hand with generalizations and apparently growing degrees of universality.

In our contribution we propose to revisit two different approaches, which take the mediation of genetic and structural aspects into account, namely that of Ernst Cassirer and that of Peter Damerow. Though they seem to represent two different approaches of "idealist" and "materialist" background, both consider historical and cognitive dimensions of the relation of abstraction and representation. Drawing on these approaches will help to focus on the question of the precise role of the symbolic means of knowledge representation in the process of externalization. If particular social conditions provide the means for abstraction and for establishing certain symbolisms, how do they shape the possibilities and constrains of application? How do more general knowledge structures then come about? How are social, material, and cognitive dimensions interrelated in the symbolism? And to what extent does their configuration condition the concrete historical developments? These questions do not only pertain to the emergence of number and mathematical practices in early civilizations, but also to any later process of abstraction. In exploring the specific attention of both approaches to abstraction as a means of establishing higher order concepts, we aim to understand the process of abstraction in a specific semiotic sense to disentangle and re-connect the political and epistemic levels involved.

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Saturday, January 18 | 14:20 – 15:00 | Parallel session 2b | → HG E 33.1
How to notate a crossing of a braid? Notation as epistemic and/or as a hindrance

Weaving, and hence braiding, can be counted as one of the oldest techniques belonging to the human culture. Yet surprisingly, only in 1926 a comprehensive mathematical theory of braids was published by Emil Artin, who tried to formulate the algebraic rules of the set of braids with the tools of group theory. Obviously, braids were researched mathematically before Artin's treatment. Alexandre Theophile Vandermonde, Carl Friedrich Gauß and Peter Guthrie Tait all attempted to introduce notations for braids. However, it was only Artin's approach that was proven successful, and that prompted a further research in the 1930s and the 1940s, even after Artin himself did not deal anymore with the subject during these years. The obvious question arises: why? One may suggest several plausible, historical explanations (the rise of group theory only at the end of the 19th century is a possible one), but I would like to propose another explanation, why Artin's method was accepted. This explanation lies in the way braids, their crossings and their deformations were notated.

This talk will analyze three case studies of notation of braids, done by Tait, Artin, and the Italian mathematician Modesto Dedò. Every one of the mathematicians had a different goal and the braids were accordingly considered by each of them in a different context. All the three mathematicians presented different notations of braids and their deformations. Hence I will attempt to answer the question: what makes a system of notation an epistemic one? As I will aim to show, the different types of notation systems either acted as a "catalyst", affording and prompting new results to be formulated, or they were operating the other way around, as a hindrance.

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Monday, January 20 | 17:40 – 18:20 | Parallel session 7a | → HG E 33.3
"Give me a lever": Bolzano and the practice of applied mathematics in Prague at the beginning of the 19th century

The general aim of our talk is to explore the interplay between changes in the institutional landscape and changes in mathematical practice by analysing the teaching and learning of mathematics in Prague in the late 18th century. In this talk, we shall focus in particular on a manuscript containing Bernard Bolzano's written examination to become professor of elementary mathematics at Prague University. This examination took place in October 1804, and consisted of a written and an oral part. Only two candidates took part to it, namely Ladislav Jandera, who won the chair of mathematics, and Bernard Bolzano, who became professor of the recently created university chair of science of religion.

The committee asked three questions to the candidates, two of which belonged to the domain of applied mathematics and the other one to the domain of pure mathematics, according to the classification of Abraham Gotthelf Kästner (Göttingen), whose textbooks were used at Prague university at the turn of 19th century. In our talk, we especially analyse Bolzano's answer to the question on the law of the lever, where he discusses several proofs and then adds his own. We shall use that answer to better understand how applied mathematics (in this case mechanics) was taught and done in Prague at that time by comparing Bolzano's answer and the approach of two of his teachers (Stanislav Wydra and Franz Joseph Gerstner) on that subject with Kästner's approach, which Bolzano himself criticises.

Additionally, this case will allow us to show how the resolution of the examination committee had an influence on the perpetuation of certain mathematical practices. That way, while Jandera does not seem to have moved far away from the practices that he inherited, Bolzano went on to propose and develop mathematical practices that hinted at ground-breaking concerns and features but which, from his position, did not have a great impact on the Czech mathematical community and Germanic mathematicians of his time.

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Monday, January 20 | 15:10 – 15:50 | Parallel session 6b | → HG E 33.1
The emergence of geometric knowledge: an interdisciplinary approach

In current debates and studies about the emergence and development of mathematical cognition, nativist approaches are one of the most active and with high academic impact. In this broad area of research, our interest focuses on the so-called geometric cognition. Some of the main proposals made by reputed cognitive scientists address the existence of a "natural geometry" (Spelke et al. 2010) and how human beings possess some innate, culture-independent geometric intuitions (Dehaene et al. 2006).
However, we think that these proposals face some conceptual problems: (i) they use a misleading vocabulary when analysing their experimental data, and (ii) they carry out an unfounded and unjustified transition from early biological and cognitive capacities to the development of mathematical knowledge. This applies both to accounts of the origins of geometry and arithmetic, but we focus exclusively on the first.
To clear up these conceptual confusions, we propose a tripartite division of our cognitive abilities related with geometric cognition. In the first level, instead of geometric cognition, we find it advisable to talk about visuo-spatial cognition. Here, we find cognitive abilities linked with the daily use of spatial environmental cues for different tasks. In the second level, we place basic 'geometric' cognition, which would be related with the emergence of proto-geometry. Apparently, to reach this level, the agents rely on using cognitive tools such as diagrams or maps to represent spatial relations. Finally, third level, we have the development of geometric knowledge properly speaking. Here we have a kind of knowledge that is guided by particular goals and values—such as problem-solving, abstraction, or generality—, and several cognitive abilities come into play, such as imagination or symbolic thought.
We argue that, contrary to nativist proposals, there is nothing like a natural geometry in the human genus. To talk about geometric, or even proto-geometric knowledge, it is always necessary to take into account the cultural environment where this kind of knowledge emerged, as well as the human concerns that may have led to the creation and utilisation of it (e.g. building construction or astronomy). In this sense, although the existence of some innate cognitive foundations could be accepted (visuo-spatial cognition), this would not imply the development of any kind of universal mathematical knowledge.

We will present a case study to illustrate our proposal. Particularly, our attention will be on the emergence of geometry in Early China, which testifies how the development of this kind of knowledge was culturally and socially influenced.
To conclude, we want to emphasize that even if cognitive sciences have been considered as an interdisciplinary field since its beginnings, history and philosophy have been usually left out from the scene. We think that this is a mistake. Cognitive studies about mathematical cognition can and should profit from analyses in philosophy of mathematical practices, and from careful historical work, in the same way that some historians and philosophers currently work embracing the inherent cognitive aspect in mathematical knowledge development.

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Tuesday, January 21 | 15:10 – 15:50 | Parallel session 9a | → HG E 33.3
Mathematical Language Processing: A conceptual and technical framework for the automatic treatment of mathematical texts

As a result of the "practical turn" in the philosophy of mathematics, a significant part of the research activity of the field consists in the analysis of all sorts of mathematical corpora. Accordingly, the problem of mathematical language (inscriptions, symbols, marks, diagrams, representations, etc.) has gained increasing importance, since decisive aspects of mathematical knowledge have been shown to be related to regularities and emergent patterns identifiable at the level of mathematical signs in texts. However, the specific problem of mathematical language and signs is often reduced to extra-linguistic features (cognitive, psychological, sociological, logical, etc.) and concrete tools available for the analysis of actual mathematical texts remain rather poor and difficult to employ without bias.

One of the main reasons for the absence of a specific and suitable theory of mathematical signs can be attributed to the fact that, even in the case of the most critical perspectives, the problem of language in mathematics continues to be governed by the highly speculative viewpoint established by the tradition of the "philosophy of language", concerned with questions such as representation, reference, truth and reality, and presupposing an insurmountable difference between natural language and the practice of mathematical signs. Moving away from this perspective, I will propose an alternative approach to the treatment of mathematical language by associating the latter to the mechanisms studied within the field of linguistics. In particular, I will outline a conceptual and methodological framework based on the latest advances in computational linguistics and their implicit renewal of the structuralist and distributionalist traditions (Gastaldi, 2019). I will argue that such an approach can 1) offer original conceptual tools to understand the systematicity of mathematical practices as the result of specific linguistic practices, following the distinction between practices, usage, norms and systems (Hjelmslev, 1936); 2) provide technical tools for the formal analysis of mathematical corpora, based on an adaptation and generalization of existing methods in computational linguistics and natural language processing, such as LSA (Landauer, 2007) and word embeddings (Mikolov et al., 2013); and 3) account for possible logical properties as emergent structures of mathematical texts, resulting from an unsupervised typing procedure based on orthogonality relations defined at a syntactic level, following (Girard, 2001; Krivine, 2001). I will support those arguments by presenting the results of a work-in-progress computational implementation of a formal model of the intended analysis and I will discuss the envisaged applications to mathematical texts based on related work in the corresponding field (Geuvers et al., 2008).

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Saturday, January 18 | 11:40 – 12:20 | Parallel session 1b | → HG E 33.1
Representations and their cognitive significance in mathematics

In this talk, I will briefly present some examples of the use of mathematical representations based on previous work on several case studies (De Toffoli & Giardino 2014, 2015, 2016; Eckes and Giardino 2018) and I will pinpoint the cognitive roles that representations have in guiding the mathematical reasoning and (in some cases) in leading to a mathematical result. In particular, I will argue that some kinds of representations have a double function, since they are at the same time the available instruments that allow exploring and modifying what they are intended to represent and mathematical objects in themselves; for this reason, they function as dynamic tools presenting particular affordances and they may be useful to provide classifications. Moreover, the case studies show that mathematical representations are in most if not all cases "heterogeneous", since they include diagrammatic as well as symbolic or textual elements. In the final part of the talk, I will try to bring all these elements together into a general framework to the aim of specifying the cognitive steps that allow going from the reconfiguration/manipulation of a singular figure or a piece of notation to the definition of some mathematical result. This framework will attempt to deal with two further and crucial issues: the relationship between external representations and imagined procedures, and the opportunity of blurring the distinction between diagrammatic and symbolic reasoning.

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Saturday, January 18 | 12:20 – 13:00 | Parallel session 1a | → HG E 33.3
The interplay between physics and mathematics: from Dirac's delta to distribution theory

Dirac (1958) introduced the delta function as a convenient method to normalize basic vectors with continuous parameters. However, given the way it is defined, such a function is inconsistent: there is simply no function which satisfies the required properties. From the physicist's point of view, the situation is not particularly worrying, since such a weird function is going to appear only as a factor in an integrand, where it can be easily eliminated. However, if such a lack of rigor can be excused in a physicist, certainly cannot in a mathematician (for more on the notions of rigour in physics and mathematics, see Urquhart 2008a, Urquhart 2008b). Given the delta function's inconsistency, mathematicians looked for a way to make sense of it without loosing its physical effectiveness. The result of such attempts is Schwartz's distribution theory, which extends the class of ordinary functions.
How is it possible that Dirac successfully applied an inconsistent mathematical theory? Some philosophers suggested that by adopting paraconsistent logic to characterize scientific practice, we can make sense of why we can use inconsistent mathematics in application to physics (see for example Mortensen 1995, Colyvan 2008a, Colyvan 2008b, Benham, Mortnensen & Priest 2014). However, this explanation only explains why we can apply inconsistent mathematics, but it does not explain why this inconsistent mathematics is effective at all.

In the present talk I will analyze in detail the role played by the effectiveness of the delta function in fostering distribution theory. I will argue that the effectiveness of the inconsistent mathematics employed by Dirac (roughly speaking, function theory plus delta function) is justified by the fact that this inconsistent mathematical theory can be embedded in a consistent, more general theory (i.e., distribution theory) which in turn is effective in representing the physical domain. This explanation builds on a previous result from (Ginammi 2016), in which it is argued that mathematical effectiveness in physics can be characterized in terms of a monomorphism from the physical target to the representing mathematical structure.
Moreover, this explanation of the effectiveness of the delta function better clarifies the role played by this effectiveness in fostering distribution theory, and the interplay between physics and mathematics: the inconsistent, effective theory motivates mathematicians to look for a more general, consistent theory; this consistent theory, in turn, is precisely what made Dirac's inconsistent theory effective, despite its being inconsistent.

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Saturday, January 18 | 11:40 – 12:20 | Parallel session 1a | → HG E 33.3
On the Cartesian significance of David Hilbert's Grundlagen der Geometrie

In Chapter 7 of the celebrated Grundlagen der Geometrie (1899), Hilbert investigates the plane geometrical constructions which can be performed on the basis of his axioms for plane Euclidean geometry, by means of suitable geometrical instruments. In this section, Hilbert proves important results on the solvability of geometrical construction problems with the aid of specific practical means, such as the ruler, the compass and, particularly, the "protractor of segments" [Streckenübertrager]. The latter instrument is a kind of marked ruler, which serves to lay off an arbitrary segment of a given length on a straight line. Hilbert shows that while every geometric construction problem which is solvable on the basis of his axiom system can be carried out with a ruler and a protractor of segments, not every "Euclidean" construction problem is solvable with such restricted geometrical means. For example, in order to solve the proposition I, 22 of Euclid's Elements, which asks to construct a triangle from any three lines which satisfy the "triangle inequality" property, the latter instrument is not sufficient, but one must resort to the use of the compass. These metatheoretical results about the solvability of a construction problem with certain geometrical means are grounded on the algebraic consideration of different sub-fields of the real numbers, such as the (minimal) Pythagorean field and the Euclidean or constructible field. In Hilbert's approach, the algebraic structures of different number fields provide a general criterion for the possibility of geometrical constructions with several practical means.

The aim of this talk is to explore the historical and philosophical significance of Hilbert' axiomatic investigations into geometric construction problems. More specifically, we will analyze the meaning of these investigations by considering them from the perspective of the central "Cartesian" program in early modern geometry, which aimed at the classification of geometrical problems according to the simplest means for their solution. As is well known, throughout La géométrie (1637), Descartes sketched a hierarchy of problems by sorting them out into classes according to the degree of their associated equations. In other words, he showed how the degree of the equation associated with a problem contained information about the constructability of the geometric problem itself, which could be also used to establish an algebra-based classification of problems.

On the one hand, we will argue that Hilbert's axiomatic investigations can be taken, from a conceptual perspective, as the accomplishment of Descartes' original geometrical program laid out in La géométrie (1637). In particular, we will claim that Hilbert's main contribution to this program consisted in providing rigorous proofs of the impossibility of solving geometric construction problems with certain restricted means. On the other hand, we will focus on the programmatic character of these investigations, for they prompted the development of a novel and fruitful mathematical theory, namely plane construction field theory.

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Mathematical Practice in Contemporary Biology: Field, Lab, Voting Booth

My current interest in the power of mathematics to organize and propel current research in biology was spurred by watching the work of my brother Ted Grosholz, a marine biologist at the University of California at Davis who studies population dynamics, community ecology, and invasive species; and the work of my friend from high school, Ruth Geyer Shaw, a population geneticist who works with one of her brothers (a mathematician, Charles Geyer) at the University of Minnesota, and among other things studies the effects of the fragmentation of the midwestern prairie on various populations of prairie plants, using the Aster Models for life history analysis that she and her brother developed. Recently, I've noticed that the work of both of them has acquired an ethical and political dimension: Ted was part of an (effective) effort to rid the San Francisco Bay of invasive grasses, and Ruth has been writing about (effective) methods to identify organisms that are under stress, and propose methods to strengthen and save them. This turn depends both on intersecting with politicians, lawyers and environmental groups, and on collecting enormous amounts of data which must then be correctly organized and processed in order to be presented to the public in a convincing way. Genbank, the NIH Genetic Sequence Data Base, is a case in point; at UC Davis, the data is filtered through the Data Science Initiative which interacts with almost all the biologists. An increasing output of information (at the level of the whole organism and at the level of the microbiome) makes processing algorithms, collation software, natural language processing and machine learning programs indispensable tools for integrating data across fields. Having visited both the mudflats of Bodega Bay, Tomales Bay and San Francisco Bay, and the praires around Minneapolis this summer, I want to give an account of the role played by mathematics in this increasing crucial research: here practice is both theoretical and, yes, practical.

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Tuesday, January 21 | 11:00 – 11:40 | Parallel session 8a | → HG E 33.3
From the group concept to group theory

Modern mathematics is abstract and axiomatic. Arguably, the most prominent example for the rise of modern mathematics is the abstract group concept. It is therefore natural to divide the history of group theory into two parts: First was a long period of gradual development of the abstract group concept in the 19th century. Once the abstract group axioms were established, group theory flourished as new autonomous branch of mathematics (around 1900), before dividing into a number of sub-branches afterwards (1920 –). A typical representative of such a reading of history is [Kleiner].

In this presentation, I will challenge this view of mathematical history. I want to present arguments, explanations and historical evidence that together form a coherent picture of a new reading of (parts of) the history of modern mathematics:

First, by outlining the general development of group theory of the last decades of the 19th century, I want to show that the adoption of the axiomatic method was not crucial for the boost group theory witnessed during that time, but rather a by-product of it.

Second, the role Felix Klein played during this development is not to be underestimated. Not only was he the main "transmitter" of group theoretic ideas from France through Germany to the USA, but also established numerous group theoretic results himself.

Third, Klein's methodology and mathematical style is sometimes dubbed as "anti-modern" [Mehrtens]. I will argue that his understanding of group theory is of unifying, intuitive ("Anschauung") and geometric character and is therefore consistent with his personal "philosophy of mathematics" [cf. Biagioli]. Group theory became successful not despite an "anti-modern" approach to it, but because of it.

Forth, I tentatively argue that the adoption of the axiomatic method represents a change of style in mathematics, and not a change of content. While group theorists of the early 20th century univocally praised Klein's group theoretic contributions [cf. Loewy, Miller], his pre-axiomatic works (whose content certainly classifies as modern group theory) became unintelligible to us.

Last, these observations question the emphasis we put on axiomatization in modern mathematics, especially in the structuralist debate. [cf. Reck & Price] Therefore, I promote a practice-based interpretation of structuralism, with a focus on (inter-)structural relations and a greater tolerance towards the possibilities of defining structures.

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Monday, January 20 | 17:40 – 18:20 | Parallel session 7b | → HG E 33.1

How the tangible gets abstracts and vice versa: Experience from classes with mathematically gifted youth

"What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics." (Freudenthal 1968, p.7)

The usage of symbols/notation to encode mathematical content is an important tool for the working mathematician. Kießwetter (2006) for instance stresses their role for overcoming limits of our working memory. These symbols and concepts themselves can become the object of mathematical study and symbolizing itself can be iterated.

We discuss the viewpoint in mathematics education of Treffers (1987). He claimed (and Freudenthal adapted from him) that there are two kinds of mathematization. First horizontal mathematization: abstracting from real world phenomena and solving real world problems with fitting mathematical tools. In contrast vertical mathematization is what was called "mathematizing mathematics" in the quote above. This includes the organization of symbols and the study of these concept abstracted from the real world.

In this talk we explain how this view allows us to design open problem fields for mathematically gifted youth, we will exemplify this approach by a sheet introducing the notion of group from real world application, which includes first group-theoretic problems.

We claim that this approach also gives an answer to the problematization of the usefulness of abstract mathematics by Wigner (1960), arguing that abstract mathematics is actually quite similar to real world-inspired mathematics.

This work is informed by our work in mathematics education with gifted pupil.

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Tuesday, January 21 | 14:30 – 15:10 | Parallel session 9a | → HG E 33.3

ML to the rescue for PMP? Using machine learning in large-scale quantitative investigations of mathematical diagrams

Recent scholarship within the philosophy of mathematical practice (PMP) has shown how diagrams play important and multi-faceted roles in mathematical research. As ex- ternal representations capable of sustaining epistemic actions, diagrams are indispensable epistemic tools in many branches of contemporary mathematics (see e.g. De Toffoli and Giardino, 2014). However, qualitative studies of mathematical practice have revealed that mathematicians sometimes consciously omit the use of diagrams in their published work, possibly because of inherited cultural values (Johansen and Misfeldt, 2016).

Preliminary investigations of the prevalence of diagrams in leading mathematics journals during the past century have revealed that the numbers and types of diagrams ebb and flow. Thus, it seems, a formalist turn during the interwar-period could seem to account for an ebb in the use of diagrams, since non-formal arguments where deliberately shunned by followers of the Hilbert-Bourbaki tradition. Yet, during the 1950s, the use of diagrams seems to increase again in a way that is not explainable by the rise of desktop typesetting software for mathematics, in particular TeX and LaTeX, since these tools were only developed later. Thus, this increase — if real — should probably be viewed as a driver, rather than as a result, of the popularity of mathematical typesetting software.

However, to substantiate and elaborate on these preliminary insights and hypotheses, a more comprehensive study of the mathematical literature than is feasible with analog methods is required. Therefore, we experiment with machine learning tools for identification of images to count diagrams in a substantially larger corpus of mathematical publications. Furthermore, we experiment with machine learning tools for classification of images to compare with more traditionally derived typologies of mathematical diagrams (Johansen, Misfeldt, and Pallavicini, 2018).

In this paper, we present our methods and tools, and we report on our experiences and results in bringing digital humanities and machine learning to the philosophy of mathematical practice.

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Tuesday, January 21 | 15:50 – 16:30 | Parallel session 9a | → HG E 33.3
How does a qualitative interview study inform the philosophy of set theory?

In the philosophy of mathematics of today, scholars show more and more interest in the practices of mathematicians. Philosophers want to understand how the mathematicians' day-to-day work looks like, for they think that the mathematical practices are relevant to their ideas on mathematics. Following this attitude, a qualitative interview study (28 interview partners) was set up to investigate how set-theoretic practices look like, in particular the mathematical work on set-theoretic independence. We tackle here the following methodological question:

How does a qualitative interview study with professional set theorists inform the philosophy of set theory?

Our answer is a systematisation of the interplay of the different disciplines. We, first, distinguish the kind of questions and the languages.

Since philosophers mostly ask non-empirical questions, but in Social Science, only empirical questions can be approached, we have to relate non-empirical questions to empirical ones. Moreover, the languages in which the disciplines are practised are distinct. We need to transfer philosophical questions into the language of Social Science, and results from Social Science into philosophical language. With regard to the languages, we even have to deal with a third discipline, that is mathematics.

In our specific framework, there is, fortunately, an intermediate concept: The concept of set-theoretic independence, which is part of all three languages: In mathematics, as a matter of fact; in philosophy as an attractive phenomenon to study since it raises deep questions about truth in mathematics; and in Social Science as a topic that underlies many set-theoretic practices. In all cases, the concept of set-theoretic independence can be explicated by the same mathematical theorems.

In contrast hereto, the understanding of ‚truth in mathematics' depends on the discipline. There is a mathematical standard account, several philosophical accounts, and an analysable notion in Social Science.

Second, we retrace the path from the results of the study to their integration in the philosophy of set theory. The set-theoretic independence phenomenon raises metaphysical questions such as: Is it possible that there are mathematical statements which are neither true nor false? An answer to that question depends on the explication of the concept of truth. However, the interview study rather provides an answer to the empirical question: Do we have empirical evidence that set theorists will accept that some set-theoretic questions are unanswerable?

We argue that empirical methods such as the sociological interview study with set theorists provide evidence for an answer to empirical questions, but not to metaphysical questions. However, they can suggest new hypotheses with respect to metaphysical questions. We see how results of the interview study can be interpreted and where are the limits of their philosophical interpretation.

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Textual proof practices in Dedekind's early theory of ideals

How can we use mathematical texts to describe practices of proving? What is the relation between concrete notation used in the text and the proof practices that the author carries out? In this talk, I will address these questions in the case of proof practices in the early version of Dedekind's ideal theory [1871].
Specifically, I focus on Dedekind's power notation, which includes the expression p^n, where p denotes a set of numbers (rather than just a number) and n denotes a positive integer. As for example Ehrhardt [2016, 2017] argues, changing the notation can enable new practices of proving; in the case of Dedekind, my study shows an interesting connection between, on the one hand, how the meaning of the notational expression p^n changes as the text progresses and, on the other hand, how Dedekind uses p^n to write proofs. The study thus suggests a relation between the semiotic behavior of the power notation and the proof practices that this notation allows.
To describe how the meaning of the expression p^n changes, I take a semiotic-analytical approach, which I adapt from Herreman [2000]. In particular, this approach does not presuppose the meaning of the expressions but instead constructs it as a part of the analysis. This approach thus enriches our understanding of how the local semiotic processes that produce meaning within a text interact with textual practices of proving.

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Aristotle, Bolzano and the Question of Pure Proofs

There has recently been an urge within philosophy of mathematics to develop a clear understanding of what mathematicians mean by seeking "pure" proofs. Many mathematicians have sought"pure" proofs of theorems, but there are different ideas about what constitutes a "pure" proof, and a clear philosophical analysis is lacking.
The different existing ideas about purity often rely on or give additional historical analyses of concerns about "purity". Aristotle and his prohibition of metabasis eis allo genos is usually taken to be the origin of concerns about purity which then was adopted by Bolzano to arithmetize analysis, leading to his proofs of the intermediate value theorem (cf. Arana 2009, Detlefsen 2008, Arana 2014, Arana & Detlefsen 2011).
Detlefsen (2008, 180), for example, explains that purity was seen by Aristotle as an ideal for proofs: "In Aristotle's view, then, purity increased epistemic quality."
Within my talk I will question this historical account. I will show that these concerns within Aristotle's work about what Detlefsen (2008, 2014) calls "purity" are not concerned with or foreshadowing what modern mathematicians regard as pure proofs and that these historical analyses rather erroneously project a modern notion into the relevant Aristotelian text. I will show that though Aristotle is concerned with topical "purity" within his widely influential analysis of scientific and mathematical knowledge, he does not see it as an ideal but the very foundation of scientific knowledge, based on his metaphysical assumptions and worldview. I argue that this cannot be seen as comparable or foreshadowing the contemporary concern about "pure" proofs.
Furthermore, I will show that this modern projection of the Aristotelian prohibition against crossing genuses is already present in Bolzano, though he does not share the same metaphysical assumptions. I will show that Bolzano uses a modern misreading of the Aristotelian prohibition against metabasis eis allo genos as an additional, rather rhetorical argument within his attack against geometrical proofs in analysis. With this revised historical account I hope to not only further enlighten the history of philosophical concerns about mathematical practice but also to contribute to a philosophical analysis of the notion of pure proofs.

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How can abstract objects of mathematics be known?

In his paper Mathematical truth (Benacerraf 1973), Paul Benacerraf argues that mathematical objects do not have causal effects on our senses and so we cannot acquire knowledge about them. In his book Mathematics as a Science of Patterns Michael Resnik claims that Benacerraf's arguments are not valid. They only show that: "in so far as realists maintain that mathematical objects are causally inert and outside space-time, they should explain how we can attain mathematical knowledge using just our ordinary faculties. I will now attempt to meet this challenge through a postulational account of the genesis of our mathematical knowledge" (Resnik 1997, p. 175). In his review (Balaguer 1999), Mark Balaguer challenged the possibility of our theories referring to abstract mathematical objects, because these objects are causally inert and thus not accessible through human capacities. The aim of the paper is to meet this challenge by developing in more detail the picture Resnik presents of the development of the language of mathematics and by stressing the continuity of reference in the course of this development. I will argue that the objects studied by mathematics are not abstract but rather ideal, and I will try to show that Wittgenstein's notion of a language game is a tool for understanding how we can have access to these ideal mathematical objects.
The explanation of how we acquire knowledge about mathematical objects is based on taking into account the instruments of symbolic and iconic representation. These instruments are real things—in the case of synthetic geometry the instrument consists of a ruler, made of wood or plastic, and a compass, made of iron, and of course a pencil and paper. When we use them, we place traces of graphite on paper. These traces are physical objects, so we can causally interact with them. Nevertheless, we deliberately subordinate our manipulation of these instruments and the interpretation of the results of these manipulations to the principles of Euclidean geometry. Thus when we erect on a straight line a perpendicular, we maintain (in accordance with Euclid's fourth postulate) that the thus obtained right angles are equal, even if strictly speaking they are not and cannot be, because the traces of graphite on paper are irregular. In other words, we use the compass, the ruler, and the pencil as elements of a language game obeying certain rules. Thanks to the subordination of the rules of the language game to the principles of Euclidean geometry the instrumental practice of the ruler and compass construction allows us to study the properties of geometric objects. The objects that we study are ideal points, straight lines, and circles.

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David Hume and the Limits of Mathematical Reason

David Hume had little interest in mathematics, and yet he devoted a long section of his Treatise of Human Nature to an attempt to refute the indivisibility of space and time. In the later Inquiry into Human Understanding, he ridiculed the doctrine of infinitesimals and the paradox of the angle of contact between a circle and a tangent. Following up Hume's mathematical references reveals the role that precisely these mathematical examples played in the work of philosophers who (like Hume) were not otherwise interested in mathematics, and who used them to argue for either fideist or sceptical conclusions. That is to say, this handful of paradoxes were taken to mark the limit of rational mathematical enquiry, beyond which human thought should either fall silent or surrender to religious faith. This argument occurs, for example, in Malezieu's Éléments de Géometrie, to which Hume refers indirectly in the Treatise. It prefigures Kant's argument in the antinomies of pure reason (in the second part of the Critique of Pure Reason). The fact that it was the same pair of examples turning up in extra-mathematical or elementary mathematical writing, without ever including obvious candidates such as Torricelli's horn of plenty, indicates that they constituted a stable unit of discourse that was reproduced without further reference to mathematical literature or expertise—a meme. Hume did not seem to appreciate that while bringing rigour to the differential and integral calculus was a central problem for mathematics, the angle of contact was (by his time) a non-problem that arose in the first place only owing to the authority of Euclid. This suggests that in Hume's lifetime Euclid's Elements remained canonical, but in an important sense ceased to be authoritative. Following Hume's mathematical sources thereby shows us something about the role and significance of mathematics in the wider intellectual culture of his time. It raises comparable questions about the cultural significance of apparently paradoxical mathematical results today.

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Charles S. Peirce on Identity: From algebra to diagrams

Charles S. Peirce's conception of mathematics was quite idiosyncratic and defies simple categorization. He had a minor and secondary participation in the foundational discussion of the turn of the 20th Century, and it has been considered rather antifoundationalist (see v. g. Pietarinen 2010). In any case, many of his ideas are very close to the spirit of the philosophy of mathematical practice. He applied his semiotic approach to the analysis of historical cases in geometry and analysis (among others mathematical subjects) and discussed the rising areas of topology and theory of graphs.

An interesting example of Peirce's approach to mathematics and logic is the notion of identity as it was essential to algebra, constituting the basis of equations. As a logical notion, identity deserved a mathematical study, since "the methods of reasoning by which the truths of logic are established must be mathematical" (Peirce NEM IV, p. 19). But, according to Peirce, "mathematical reasoning is diagrammatic. This is as true of algebra as of geometry" (CP 5.148). Given that diagrams fall into the category of icons, Peirce fostered a semiotic study of mathematics. In his algebra of relatives, Peirce defined identity as a second-order predicate (see CP 3.398), but later he stressed its diagrammatic nature: "This equation is a diagram of the form of the relation" (CP 4.530). From 1896 on, Peirce formulated identity in his diagrammatic logic, the Existential Graphs. Here, identity and Quantifiers are expressed on the basis of the same sign: the line of identity of the Beta Graphs. Hence, identity is built into quantification. The aim of this paper is to examine the diagrammatic expression of identity and identity statements in the Beta Graphs, stressing its notational features (in order to see it as a problem in what Peirce called "philosophy of notation") and putting it in the context of his theory of signs (or semiotics). It will be claimed that Peirce carried out by the Beta Graphs an analysis of identity in the special sense of uniqueness of decomposition (see Bellucci and Pietarinen 2016, p. 210). Hence, the case of identity exemplifies the analytic feature of diagrams beyond their operational and structural features and promotes further discussion on the role of diagrams in mathematics.

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Sunday, January 19 | 09:00 – 10:30 | Plenary Session | → HG E 5
Pluralities and sets in mathematical practice

Philosophers and logicians have recently taken a great interest in plurals, often seeking to apply the expressive resources of plurals to mathematics and its philosophy. What is the relation between pluralities and sets? This talk will pay special attention to how mathematical practice bears on this question, including (1) Cantor's appeal to plural to explain the notion of a set and (2) a liberal view of mathematical definitions, also espoused by Cantor, which entails that every plurality defines a set. This liberal view requires us to replace the traditional logic of plurals with a more "critical" plural logic.

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Monday, January 20 | 09:00 – 10:30 | Plenary Session | → HG E 5
Mathematics or moonshine: non-Euclidean geometry in The Monist at the beginning of the twentieth century

The Monist began publication in 1890 as a journal "devoted to the philosophy of science" and dedicated to bringing European (particularly German) texts to American readers. From the first volume, The Monist regularly featured mathematical content. Many of the regular contributors considered themselves knowledgeable amateurs, and published alongside turn-of-the-century greats such as Poincaré, Hilbert, and Veblen. The mathematical content varied from recreations to the logical foundations, but everyone had something to say about the recent changes in geometry. On one side, George Bruce Halsted ceaselessly advocated the "epoch-making" role of Lobachevsky, Bolyai, and their successors. At the other extreme, a consensus that included lawyers, reverends, philosophers and less cosmopolitan mathematicians questioned the idea

that straight lines should be anything other than visibly straight.
Nineteenth-century debates around non-Euclidean geometry are well-known within the history of continental and British mathematics. Complementing these studies, a focus on The Monist reflects the particular nationalism of the United States at a time when its academic hierarchy was still in flux and mathematical research was just beginning to be recognized abroad. Philosophical arguments navigated a delicate balance between the emerging philosophy of pragmatism and the danger of mysticism. Despite ad hominem attacks and name-calling, these exchanges document deeper debates around the relationship between the scientific method and mathematics, and the role of authority (particularly foreign authorities) in shaping the future of geometry. As one contributor inquired "how is the professional expert better fitted to see more lucidly in dealing with the elements of geometry than any other person of good geometric faculty?"

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Saturday, January 18 | 12:20 – 13:00 | Parallel session 1b | → HG E 33.1
Diagrams and Figures in Ancient Mathematics: China and the West

The paradigm of ancient Greek mathematical practice is the Euclidean demonstration, for instance, that of the Pythagorean Theorem in Proposition I.47 of the Elements. Ancient Chinese mathematicians also proved this theorem, or at least an analogous one, that which goes by the name Gou-Gu, Base-Height. Like the Euclidean demonstration, the Chinese proof essentially involves a non-textual element, something we might, loosely speaking, refer to as a diagram. This non-textual element serves, however, in a crucially different mathematical way in Chinese mathematical practice from the way the diagrams we find in Euclid serve. Nor, I argue, is the proof itself a proof of precisely what Euclid establishes in the Elements Proposition I.47. Whereas what Euclid demonstrates belongs to geometry, Guo-Gu belongs to algebra. Correspondingly, whereas Euclid's demonstration employs a diagram, strictly speaking, the Chinese mathematician's demonstration appeals to what I will call a figure. The two mathematical practices, ancient Greek and ancient Chinese, differ, then, both in their subject matter, geometry and algebra, respectively, and in their method, diagrammatic and figurative, respectively. As I will indicate, understanding these differences is the first step on the way to a fuller appreciation of the nature and role of mathematical practice not only in ancient Greece and ancient China but in the larger intellectual cultures to which they would subsequently contribute, and in the case of the Greeks, partly shape.

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Mathematical "Error" in Descartes: Failure in Algorithmic-Exploratory Practice

The intellectual enthusiasms that drove Descartes' career burst out during his stay at the military academy in Breda in 1618 – 19. They centered on discoveries in geometrising physics, and then combining algebra and geometry: in March 1619, he wrote Beekman of his geometrical solutions to cubic equations, conquering new territory beyond the quadratics treated in Clavius. What then led Descartes to also focus on the problem of intellectual error in the following Winter, just when making great geometrical progress using algebra?

This confluence of intellectual success and doubt was likely triggered when he was confronted in Germany with the Italian work on cubic equations: once understood, it shows Descartes clueless! To dismiss the episode as merely an ignorant beginner making mistakes, however, is to miss important historical and philosophical opportunities. Descartes is here on the cusp of transition in Algebra between the rule-based Cossic tradition and the assertive-form based tradition he was to popularize. Philosophically, his mathematical missteps raise the issue of the nature of failure in contexts where standards are not applicable or available: where rules are not inherently "truth-directed", or where one is extending beyond established rules, or where in principle existing correctness standards are just not of convenient explicit application. Descartes' situation combines elements of all these. Some of the more creative moments in mathematical history—and contrary to orthodox philosophy of mathematics, also in current mathematics—share such characteristics.

Using Descartes' case, I reflect on the nature of correctness standards in mathematics beyond the simply-truthable. New light is also shed hereby on Descartes' own early philosophical take-aways in his "Rules for the Direction of the Mind".

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Calendrical Reform and Functionalism: Engagement of Mathematical Astronomers in Executive Practices in Early Islamic Time

This paper will focus on the mathematical practice of a Muslim astronomer in the 9th century, Muḥammad ibn Kathīr al-Farghānī. Being characterized largely for his theoretical compositions, his name also appears at two construction projects: a Nilometer in Cairo, and a water-supply canal for a newly founded city of Jaʿfariyya. However, his engineering activities gave him nothing but disgrace in contrast to the splendour of his scientific writings. This disproportionate amount of success in his scholarly life is what the present study is centred around.

Historically, al-Farghānī's flourishment was indebted to his courtly commissions at Abbasid caliph al-Mutawakkil (r. 847-861) when grave concerns were raised on the deviation of the Arabic lunar calendar from the solar agricultural cycles. Matters of taxation added to the significance of this issue as land tax should be paid in line with the lunar fiscal year, which did not coincide with the cultivators' seasonal works and resulted in growing dissatisfaction with the government. Despite being opposed to traditional Islamic law, a calendrical reform was ordered by caliph al-Mutawakkil who commissioned astronomers to work out accurate intercalary days for a lunar fiscal year to match the seasons. Backed by the caliphal authority, in the process, mathematical astronomers acquired a pivotal executive position varying from pursuing pure computational practices to construction projects. This multifaceted cooperation is well reflected in case of al-Farghānī whose contributions were not restricted to the well known calendrical calculations in his Elements of Astronomy, but rather manifested in his nomination for two practical functions. First, we find his name as a collaborator in renovating the Great Nilometer in Cairo, eventhough his engagement was limited to the role of one of the supervisors. A Nilometer is a structure of measuring the water level in the river Nile during annual inundations as a decisive factor in determining the ratio of land tax with which al-Farghānī's association can be explained based on his expertise in chronology. Getting involved in this project, he felt confident enough to undertake solely the engineering of a water canal for the caliph's best-loved city of Jaʿfariyya which ended up in a dishonourable failure resulted from his erroneous assumption of the depth of the canal mouth.

This historical evidence provides us with a case where apparently theoretically oriented scholars, engaged in mathematical practices, often working in the interface between the necessities of computational prediction and de facto accuracy. This tension manifested in the emergent role of the mathematician as a civic functionary. Even though inconsistency of the calendars with administrative requirements called for the attention of the courts to the importance of precise computations, it did not bring about a systematic collaboration among scientific and executive agents. In other words, while political patronage provisionally drew mathematical scholars to practical projects in early Medieval Islam, how did this relationship emerge in the practice of mathematics and of state building at the same time, is the question this paper would seek to address.

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Monday, January 20 | 15:10 – 15:50 | Parallel session 6a | → HG E 33.3
A Copernican Revolution in the Lagoon: When A Galilean Mathematician Tried to Solve the Hydrogeological Problems of Venice

The work On the Measurement of Running Water (Della misura dell'acque correnti, 16291) by Galileo's pupil, Benedetto Castelli has been considered one of the foundational works of modern hydrodynamics. It offered geometrical demonstrations aimed to make the measurement of running waters (the "misura") possible through the isolation of few variables: the section of a waterway and its velocity. From this viewpoint, Castelli's work represented another 'Galilean' attempt at mathematization. However, Castelli was not able to convince the Venetian authorities that his method was apt to solve the main problems related to the conservation of the geoenvironmental equilibrium of the lagoon. On the one hand, the Venetian authorities saw the diversion of rivers outside the lagoon as a measure to mitigate the infilling of sediment; on the other, Castelli argued, to the contrary, that it was precisely rivers' diversion that produced an embankment effect, because it drove away a great quantity of water, which he accurately calculated. His computational approach was dismissive of the comprehensive knowledge and complex methods that Venetian water experts had developed towards a systemic understanding of the hydrogeology and the environment of the lagoon. They took into account manifold factors as varied as the rivers' flows, sea tides, the relative positions of the sun and the moon, winds, and even the effects of anthropic interventions. The dryness of Castelli's reductionist approach, bolstered by his mathematical modeling of running water, was received with skepticism, even rage, thus rejected, in spite of the prestige of his connection with Galileo.

We reconstruct this controversy to dwell into the tension between mathematical abstraction and its claims to prescribe solutions to problems of the physical world, sparked off by Castelli's claim that his mathematical treatment of running waters could solve all of the most urgent problems linked to the management of the Lagoon of Venice. From an epistemological viewpoint, we ask, to what extent it brought about a conflict between physico-mathematical abstraction (which resulted from the isolation of particular variables to yield a set of quantifiable data) against 'geological' concreteness (a form of comprehensive knowledge aimed to cope with systemic complexity). We will here consider whether the two different approaches were rooted in different societal arrangements and corresponding scientific practices, resulting in different modes of abstraction in practice.

To summarize, from the viewpoint of the philosophy of mathematical practice, this communication will explore a Renaissance case showing:

a the practical use of the Euclidean theory of proportions;
b its prescriptive function as a means of mathematical abstraction in engineering;
c the politics behind such mathematical assessments, forms of expertise, andmanagement of physical resources and their political management through technical prescriptions, tested in practice.

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Monday, January 20 | 12:20 – 13:00 | Parallel session 5b | → HG E 33.1
Grounding mathematical concepts in practices: language games and conceptual development

During the last few decades, philosophy of mathematics has turned to the notion of practice for developing an analysis of mathematics as a social and historically situated activity. But despite the progress made in this direction, philosophers have not yet agreed on how to relate mathematical concepts to actual practices.
Building on Wittgenstein's (1958) use theory of meaning and Toulmin's (1960, 1969, 1972) theory of concepts; we propose a semantic framework that could solve this problem. The "later" Wittgenstein famously argued that concepts gain their meanings embedded in (rule-governed) language games which are "grounded" in different behavioral and practical contexts (forms of life). Following these ideas, Toulmin claimed that scientific concepts cannot be analyzed in abstracto, because they have a stratified nature. That is, their contents are the products of the evolution of sequences of language games, which are, at the same time, associated with different culturally situated collective practices. In this sense, analyzing the content of a scientific concept imply to look into its developmental history and, more specifically, to look into the collective practices that constitute the language games in which it was involved.

We will apply these ideas to mathematics, taking as a case study the mathematization of perspective in the 16th century in the work of Guidobaldo del Monte. In particular, we will show how the Italian mathematician developed the first proof of the vanishing point theorem by incorporating a technical rule from perspective drawing—the convergence rule (Andersen 2007)—into a mathematical setting. This process of mathematization, that was a crucial step for the formalization of the notion of vanishing point, was the result of the interplay of several pre-existing language games, corresponding to different scientific and non-scientific practices (perspective drawing, euclidian geometry and optics). In this sense, this episode illustrates Toulmin's point about the relationship between language, concepts and practices; while it offers an interesting insight on the pragmatic dimension of some mathematical concepts.

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Notational bearings on conceptions of proofs

In his 1934 – 1935 thesis (Gentzen, 1934 – 35), Gentzen introduced two types of proof systems, namely systems of natural deduction and sequent calculi. Gentzen took great pains to show how his systems corresponded and were related to the axiomatic systems of his predecessors like Lukasiewicz, Hilbert, Heyting, etc. Two important results presented in the last two chapters of the thesis are a proof of equivalence of three intuitionistic calculi, namely LHJ, NJ, and LJ, and also a proof of equivalence of three classical calculi, LHK, NK, and LK. The first proof consists in 1) a transformation of LHJ, an axiomatic calculus, into NJ, a natural deduction calculus, 2) a transformation of NJ into LJ, a sequent calculus, and finally 3) a transformation of NJ back into LHJ. The proof for the classical calculi runs along similar transformations. Although these transformations establish the equivalence of calculi, Gentzen was not interested in designing superficially different notational variants. Indeed, Gentzen thought that natural deduction systems and sequent calculi could be regarded as improvements over axiomatic systems, in that they seem to better reflect our ordinary, human way of reasoning. For one thing, these systems do not have to rely on the notion of truth in their formulation: we can correctly reason without knowing the truth-value of the content we reason with and from. For another, and this is perhaps the most important improvement, natural deduction systems can accommodate the idea that we reason from assumptions, and not from sentences of a given form. Moreover, natural deduction and especially sequent calculi seem to be better suited than axiomatic systems to the study of the structure of deduction, what Prawitz (1974) later labelled "general proof theory". This raises the question: how can systems of natural deduction and sequent calculi, which are in a sense equivalent to axiomatic systems, nevertheless depart from them in non trivial ways?

In this paper, I argue that despite their shared equivalence with axiomatic systems, the calculi introduced by Gentzen give us two different ways of reasoning with hypothetical proofs. I further argue that these two views of hypothetical proofs correspond with specific features of the notations. I first discuss different conceptions of hypothetical judgments and their relation with assumptions and assertions. More specifically, I address how they are associated with different interpretations of propositions and sequents. I then show how what Schroeder-Heister (2016) calls the "no-assumption view", the "placeholder view of assumptions", and "biderectionality", find their natural home in axiomatic calculi, natural deduction, and sequent calculi, respectively.

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Recipes for talking about mathematical progress

If foundational issues monopolized the philosophy of mathematics at the turn of the 20th century, its focus has now successfully been broadened to include investigation into the practices of real-life mathematicians. While this is certainly a welcome change, there are cases where the notion of practice appears ill-suited to characterize the actions of a mathematical agent. In particular, when dealing with issues related to mathematical progress and innovation, the notion of practice seems to obfuscate the contribution of individual mathematicians to the generation of novel content. The problem is that practices are typically held to be social structures composed of repeated performances of the same actions (e.g. Rouse 2006), whereas progress changes such structures and adds new elements to them. And yet, to explain progress and evolution in mathematics, we have to have an account of the processes responsible for the generation of novel methods and new theorems.
While it is tempiting to appeal to the well-known parallels between biological and cultural evolution (e.g. Richerson & Boyd 2005; Mesoudi et al 2006) to find a mechanism capable of generating novel practices, in this talk, I argue that accouts of cultural evolution do not have a specific process equivalent to genetic mutation that could serve to explain where novelty comes from. I then explore the potential advantages of using the notion of cultural recipes (e.g. Charbonneau 2016; Schiffer and Skibo 1987) instead of that of practices in order to better factor in the role of the innovative individual in the generation of novelty in mathematics. I then close by applying this recipe-based description of mathematical progress to determine to which extent adopting a radical enactivist perspective to numerical cognition (Hutto 2019) can reflect the important role played by a cultural niche populated by experts and cognitive tools in modifying cultural recipes during episodes of mathematical progress. I argue that radical enactivists do not have the conceptual resources to account for all cases of mathematical progress, since their organism-centered views make it difficult to frame the development multi-agent recipes such as computer-assisted proofs or proofs that require multiple mathematicians to complete or verify.

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Mathematical modelling and teleology in biology

Teleology (telos: end, goal, purpose; logos: reason, explanation) is an explanatory strategy that appeals to the purpose of the object of study, rather than its mechanical causes. Biology has traditionally incorporated not only mechanical explanations, but also, teleological explanations. Yet, even modern biology, far away from vitalism and intelligent design, still includes teleological notions in its explanations either as metaphysical propositions or at least as a heuristic strategy, acting "as if" biological phenomena were subjected to design or had purposes (Ratzsch, 2010). It is because of these non-mechanical components in the explanations of biology that it has been proposed to be irreducible to strictly mechanistic sciences such as physics (Ayala, 1968; 1999). Furthermore, it has been argued that the teleological component of biological explanations cannot be eliminated without loss of information and explanatory power (Ayala, 1999).
However, it is believed that the mathematization of biology is progressively putting it in line with the standards of rigor of the physical sciences, and that this is the way to go. Yourgrau and Madelstam (1960) claim that teleology is reflected in natural language, not in mathematical formulas. Indeed, formulas can describe the motion of the rock, but not its purpose. Enquist and Stark (2007) fully endorse the development of a "quantitative, mechanistic and predictive biology" so that it becomes a "capital-S Science". It is a popular idea among scientists and philosophers is that the more mathematical a science is, the more mature and rigorous it is (Storer, 1967).

Mathematical modelling is a group of techniques which have been making their way into diverse biological fields. The incipient roles of these techniques in biology are transforming the scientific practice, but not exactly as outlined above. In this talk I will challenge the idea that mathematics brings biology closer to the standards of physics by showing how teleological notions, common in biology but not in today's physics, coexist and interact with modelling techniques in a very idiosyncratic scientific practice. To this end, I will explore modelling techniques of the so-called brain's "internal compass", a component of the "brain GPS system", in computational neuroscience. Understanding how scientists of different backgrounds use mathematics to represent phenomena is a prerequisite for a proper epistemological understanding of mathematical modeling.

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Sunday, January 19 | 12:20 – 13:00 | Parallel session 4a | → HG E 33.3

Geometrical Practice between Unification and Purity of Methods. A 19th century case study

In this talk the current debate about unification and purity of methods will be applied to developments 19th century geometry (which is, in some respect, opposed to developments in 17th century geometry).

I will, firstly, sketch three significant developments in geometry: (a) the introduction of algebraic methods to geometry by Descartes, (b) Poncelt's opposed goal to ban all algebraic notions from geometry, which was most successfully achieved by van Staudt and (c) the geometry of the 19th century mathematician Plücker, who used analytic methods, but took over the aim of synthetic geometry to uncover the nature of geometric reasoning.

Secondly, I will show, which notions of unification and purity of methods can be best applied to the developments in 19th century geometry. It will be argued that this notions have to be based on Mander's (1989) notion of unification by domain extension and Arana's (2008) and Baldwin's (2018) notion of topical purity.

Thirdly, I will discuss the epistemic significance attributed to unification and purity of methods. It will be argued that unification provides a tool for verification, since it allows metamathematical considerations and provides a shared proof standard (Maddy (2017)), and that purity of methods provides a tool for clarification, since it shows, why a proof holds (Arana (2017) and Baldwin (2018)). It will be shown, how this applies to Descartes (in respect to unification) and van Staudt (in respect to purity of methods).

Finally, I will show that Plücker's combination of analytic methods with the synthetic geometers aim to uncover the specific nature of geometrical reasoning allowed him to achieve both a tool for verification and clarification. Therefor Plücker's homogeneous coordinates are essential. This coordinates do not express the distance of points to the origin of the coordinate system but the distance ratio to the lines of a coordinate triangle. In this way he introduces algebraic methods to geometry but he also represents the particularity of geometrical objects and reasoning methods: Geometrical objects (points and lines) can, unlike numbers, only be identified in relation to other geometrical objects. Klein (1926) therefor wrote that in Plücker's geometry "analytic operations are led back the geometric." In this talk it will be tried to spell this out more precisely.

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Tuesday, January 21 | 12:20 – 13:00 | Parallel session 8b | → HG E 33.1

How to Frame a Mathematician: Modelling the Cognitive Background of Proofs

Frames are a concept in knowledge representation that explains how the receiver, using background information, completes the information conveyed by the sender. This concept is used in different disciplines, most notably in cognitive linguistics and artificial intelligence.

We argue that frames can serve as the basis for describing mathematical proofs. The usefulness of the concept is illustrated by giving a partial formalisation of proof frames, specifically focusing on induction proofs, and relevant parts of the mathematical theory within which the proofs are conducted; for the latter, we look at natural numbers and trees specifically.

Why would one want to use frames for modelling mathematical proofs? We have three arguments which also connect our concept to related work in different areas of formal mathematics and philosophy.

First, using frames is not as unheard of as it may seem: The schemata and tactics underlying semi-automatic provers such as Coq and Isabelle can be seen as frames. These schemata also contain patterns of proofs that are partially filled in by the mathematician conducting the proof and are partially completed by heuristics implemented in the provers.

Secondly, for comprehending and checking proofs, mathematicians at least to some extent complete the informal proofs in mathematical texts using their expertise and the proof schemata they have acquired in their mathematical life. A common task for students in a BA theses is to enrich a given published (expert) proof with information that completes the proof schemata and hence shows that they comprehended the proof. Even the task to find a proof appears to be closely related to frames. There are different proof techniques which might be triggered by a domain of discourse or known partial results.

Thirdly, in the philosophy of mathematics, it is also discussed how to deal with gaps. Azzouni for instance introduces the derivation-indicator view (see Azzouni 2004, 2009), arguing that while in the daily life, a mathematician deals with proofs, those proofs are indicative of underlying derivations. Carl and Koepke (2010) argue that this view might be deeply linked with our first motivating point.

This talk presents joint work with Bernhard Fisseni, Bernhard Schröder and Martin Schmitt.

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Sunday, January 19 | 11:00 –11:40 | Parallel session 4b | → HG E 33.1
What the study of notations can tell us about mathematical practice

The design of an effective notation is an integral part of mathematical practice. However, what exactly counts as an "effective" or "good" notation has been the topic of many debates (see, for example, the famous dispute between Leibniz and Newton), and to this day, no consensus seems to have been reached. In this talk, I will focus on notations for logic and present some of the criteria by which one might want to evaluate a notation that have been put forward by leading 19th century mathematicians, such as Babbage (1830), Boole (1854), Frege (1879), Venn (1881), and Peirce (1885). Two insights will emerge from this discussion: First, the exact nature of the subject matter that a notation is designed to represent has not always been clear in advance. Instead, it seems that the design of a notation in part also contributes to establishing the subject matter in the first place. In particular, those distinctions that are represented are deemed to be the relevant ones (e.g., between "A and B" and "B and A"), while others are neglected (e.g., between "A and B" and "A but B"). This issue arises in particular in Boole's reflections about the relation between the algebra of logic and ordinary language, Venn's discussion of the nature of propositions, and in Peirce's use of existential graphs. The second insight is based on the observation that some of the criteria for good notations that have been suggested are incompatible with each other. Thus, that what counts as "good" can only be evaluated relative to a particular purpose or goal one wants to achieve with the representation. These goals can be foundational (e.g., finding the basic building blocks of reasoning), theoretical (e.g., minimizing the number of symbols), practical (e.g., allowing for easy inferences), or pedagogical and

cognitive (e.g., easy to learn and memorize). In conclusion, this historical study leads the view that notations are not merely representations of a given subject, but they play an important role in establishing the subject matter of an investigation, and that they reflect the particular aims of such an investigation. Thus, by studying notations we learn about the subject matter and the goals of mathematical investigations.

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Monday, January 20 | 11:40 –12:20 | Parallel session 5b | → HG E 33.1
Two ways to mathematical objectivity: how to salvage a philosopher's insight?

A phenomenological methodology in philosophy of mathematics is bound to approach the subject through the lens of practice, although it does not seem like the two intellectual fields have properly met. The first thing we hope to do is therefore to present in particular the theory known as "metaconstructivism", elaborated by Jean-Michel Salanskis (2008), and the ideas of two modes or ways of mathematical objectivity in the post-Hilbertian contemporary context of mathematical practice: that is, "correlational" objectivity, obtained through the consideration of the abstract "multiplicity" hypothetically given by an axiomatic system, and "constructive" objectivity given in formal and constructive rules an, though still not empirical, immediately accessible to the mathematical practitioner. This theory, however, remains entirely elaborated in the context of foundational systems and foundational problems, and restricted to post-Hilbertian mathematics, in the way typical of traditional 20th century philosophy of mathematics. A double challenge therefore arises to anyone who wishes to keep the Philosopher's insights in the context of "real", ordinary, mathematical practice, potentially belonging to earlier eras. We would therefore like to propose ways to gain in extension and lose in specificity, without abandoning the key insights that makes the theory valuable in the first place. One of those insights is that objects obtained through "correlative" (we would say "fictional/ideal" objectivity may not need to be guaranteed a collective "belief" in their "existence" to be considered as objects (and may in fact be considered entirely fictitious), as long as the "fictional/ideal" practice is supported by the collective recognition of "constructive/ideal" objects that cannot admit skepticism in the context of a given mathematical community. We propose to try and apply this approach in particular to "infinitary" practices in Ancient Greek, and, if there is time, Early Modern European, mathematical texts. Another precious insight is the idea that constructive gestures, while they cannot be deemed empirical, are still gestures, and should therefore, phenomenologically speaking, be considered as ideal gestures accomplished by an "ideal body". We propose to, again, consider whether this idea of the set of gestures permitted to the ideal body can be tied in with the general idea of the "toolbox" elaborated by Ken Saito in the context of Ancient Greek mathematics.

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Saturday, January 18 | 09:00 – 10:30 | Opening and Plenary Session | → HG E 5
Diagrams and computers in the proof of the Four-Color Theorem

The use of diagrams and the use of computers are two significant themes within the philosophy of mathematical practice. Although case studies concerning the former are abundant—from the notorious case of Euclidean geometry to the uses of diagrams within arithmetic, analysis, topology, knot theory, and even Frege's Begriffschrift—, the latter has received less attention in the field. When it is considered, the famous case of the Four-Color Theorem (4CT) is usually mentioned. I show in my talk how the two themes—diagrams and computers—can be investigated simultaneously via an analysis of the 4CT proof. I will present the roles played by the more than 3000 diagrams and the specificities of the computational machinery mobilized in the first version of the proof (Appel & Haken 1977 and Appel, Haken & Koch 1977). By exploring the main lines of articulation between diagrams and computers in this notorious mathematical result, I will propose some criteria for discussing the identity of different versions of the 4CT proof (mainly Roberston et.al 1997 and Gonthier 2005).

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Saturday, January 18 | 17:40 – 18:20 | Parallel session 3b | → ML F 38
Visual computer experiments in mathematics—interpretations and philosophical issues

Experiments of various sorts have always been an important part of doing mathematics. In the recent years, mathematical practice has been increasingly changed by computers, which in particular enabled new ways to experiment with and explore mathematical objects. In my talk I would like to raise some philosophical issues connected with the use in mathematical practice of "visual computer experiments".

In the first part of my talk I will address some general issues, in particular explication of what "experiment in mathematics" actually means. It could be broadly understood as any trial-and-error manipulation with symbols, trying out of different transformations and methods or gathering evidence for a general statement by analysing particular cases. In that sense a big portion of mathematical practice is about experimentation. Authors of "Mathematics by Experiment: Plausible Reasoning in the 21st century" in turn suggest that mathematical experiments can be conceived as transmission of insight, exploration of conjectures and more informal beliefs and a careful analysis of the data acquired. However, some further question arise, for example, how should one understand the difference between computer and paper-and-pencil experiments, and in particular, experiments with use of visualisation and without it? Trying to address those questions in my talk, I will use some distinctions, in particular one between three types of visual computer experiments. First kind involves using computation to produce computer graphics of a type that is usually associated with a given branch of

mathematics—for example geometric and topological objects or graphs. Secondly, one can point at visualisations that are non-standard for a given mathematical concept and origin in some creative idea to represent in 2-dimensions concepts that are not inherently spatial. This could be a visual representation of number-theoretic relations or in fact the famous Mandelbrot's set which originated in a clever on novel idea of representing facts about series of complex numbers as points in a plane. The third type of frequently used computer visualisation are representations such as scatterplots or line plots which are typical of data analysis. Their use aims at providing better insight into large amounts of data that is generated using computers.

Finally, I will turn to interpretation of what visual computer experiments are within the philosophy of mathematics. Some philosophers and mathematicians have suggested that aspects of visual computer experiments support realism in mathematics, or at least the thesis that mathematical truth is objective. On the other hand, analogies can be drawn between experiments in mathematics and the natural sciences in terms of the quasi-inductive methodology used and inexactness of the results. But in what sense exactly are computer visual experiments compatible or in conflict with realism, quasi-empiricism or other standpoints in the philosophy of mathematics? Answering those questions will require, among others, understanding how "experiment" is to be understood in the philosophical context: what are the "data" analysed, what is the subject and outcome of an experiment? Finally, how exactly does it differ from experiment in the natural sciences?

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Monday, January 20 | 14:30 – 15:10 | Parallel session 6b | → HG E 33.1
The BMI and mathematical practice: Abel's exception, history of infinity and cognitive accounts of mathematics

Over the past two decades, cognitivist accounts have provided us with new insights and perspectives on the embodied foundations of basic arithmetic and how these can lead to higher mathematics. A cornerstone in this process is the so-called Basic Metaphor of Infinity (BMI). In order to argue for the existence and influence of the BMI, proponents of the cognitivist account have invoked arguments drawn from the history of mathematics. Yet, despite their claim to propose a naturalistic philosophy of mathematics, their use of historical evidence tends to be reductionistic in a way that severely undermines their argument.

Therefore, based on a more nuanced analysis of the role of arguments involving infinity in the theory of series from Leonhard Euler in the mid-18th century via Niels Henrik Abel and Augustin-Louis Cauchy to Karl Weierstrass in the late-19th century, this paper argues that history of mathematics has a more profound role to play in understanding the BMI and the construction and application of conceptual metaphors more generally. In particular, we identify three distinct uses of intuitions about infinity in the research and foundations of the theory of series in the long 19th century: 1) the blend of the finite domain as a completed series generally prevalent in Euler's work and exemplified in his series expansions of the exponential function, 2) the critical revision at the hands of Cauchy and Abel who came to realise that criteria were required for the blend to work, in particular when dealing with trigonometric series, and finally 3) the complete autonomy of the infinite domain exemplified by Weierstrass' construction of a Monster breaking all the finitely grounded intuitions about infinite series. Through 'thick' reconsideration of these instances, we show how choices and negotiations played important roles in shaping the Basic Metaphor of Infinity. Thus, we argue that an empirically grounded, historically informed cognitivist account of mathematics cannot ignore social and contextual components to the degree that the account of the BMI has done.

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Monday, January 20 | 15:50 – 16:30 | Parallel session 6a | → HG E 33.3
The Map of France and the Shape of the Earth: the Eighteenth-Century Debate over Cartography, Mathematical Practices and Cosmology in the Paris Academy

The realization of an accurate map of France was a central concern for the Paris Academy of Sciences in the seventeenth and eighteenth centuries. As of the 1670s, illustrious astronomers and cartographers working at the Academy are encouraged to undertake operations of measurements of the meridian arc running through the country. Elaborating on the results of the operations conducted in France until 1713, the leading cartographer of the Academy Jacques Cassini elaborated a first sketch of a map of the kingdom, but also drew a general conclusion on the shape of the Earth. The Earth, according to Cassini, is elongated toward the poles. This conclusion is at odds with what Newton states in the third book of the Principia (1687), namely that the Earth must be slightly flattened at the poles for the combined effect of attraction and of the daily rotation around its axis. The presence of Newtonian scientists in the Paris Academy triggered a long-lasting debate over the shape of the Earth, where disagreement over technical issues went together with the opposition on cosmological stakes. The Newtonians questioned Cassini's cartographical practice based on the lack of accuracy of the instruments he used and of the astronomical observations that served as a basis for his operations. Second, the Newtonians attacked the poorness of the mathematical tools Cassini employed: he was not acquainted with calculus, and could not therefore elaborate equations to make the determination of the meridian arc easier and more accurate. Third, the cosmological framework of Cassini's narrative appeared questionable, insofar as he held a Cartesian perspective closed to any possible influence from the Newtonian tradition. In order to come to an accurate determination of the meridian and the shape of the Earth, two expeditions led by Newtonian scientists were sent to Ecuador and Finland to measure meridian arcs across the Equator and the North Pole. The cartographical and mathematical practices at work in these expeditions were opposed to Cassini's, and so was the outcome. The results were in fact more relevant for the making of a new map of France, if not of the world, and, on the cosmological side, Newton was proved right on the Earth's flattening. In my paper, elaborating on this narrative, I will raise the following questions:

1. How did the relationship between cartographical practices, the mathematization of territory and the quest for cosmological truth emerge, not only in their articulation, but also embedded in the practice of the different expeditions?

2. How was the promise of mathematical abstraction carried out in practice between the contending arguments, qualifying norms such as accuracy, and the ease of computation as an epistemic necessity in measuring the Earth?

3. How did mathematical practices acquire their place and shaped in turn the political and military project of mapping France from within the battlegrounds of the Paris Academy of Sciences?

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Monday, January 20 | 17:00 – 17:40 | Parallel session 7b | → HG E 33.1
Exploring the efficacy of practice before instruction

"Practicing should be experiment, not drill"
— Artur Schnabel

In traditional mathematics education, students are formally instructed in, and then practice, some element of mathematics (e.g., variance). Recent empirical work indicates that a reversal of this order—first practice, then instruction—leads to improved learning outcomes (e.g., Kapur, 2014, on "productive failure"). Why would practice before instruction be more pedagogically effective than practice after instruction? This phenomenon remains an open question in the learning sciences. By presenting at APMP 2020, we hope to discuss this puzzle more widely with scholars whose expertise complements our own.

That one's practice of an established activity is regulated by experts may be a productive starting point for our discussion. This claim has its precursors in activity theory, where novice practitioners are theorized as "other-regulated" by more-knowledgeable practitioners (see Vygotsky, 1978). Recent work in cognitive neuroscience suggests that other-regulation is literal—we seem to be regulated at a neural level by our perceived authority figures (Caspar et al., 2016). Behavioral studies corroborate this finding. When told what to do, students spontaneously restrict their exploration, even if later explicitly asked to explore (Bonawitz et al., 2011). This may explain findings in education research. In mathematics education, instruction followed by practice appears efficient on measurements of rule-adherence, yet it can also result in persistent beliefs that mathematics is a rolodex of inflexible rules-to-be-followed (Trninic, Wagner & Kapur, 2018).

Practice before instruction has been put forward as a method of navigating between the devil of excessive regulation and the deep sea of inadequate guidance. But while this practice-before-instruction approach appears pedagogically effective, its efficacy is not well understood. We identify one potentially useful starting point: namely, that regulation restricts exploration, and excessive regulation turns experimentation into drill.

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On abstraction theorems for homotopy categories

Freyd proved that the homotopy category was abstract, i.e., it cannot be embedded in the category of sets by way of a faithful functor. In particular, its objects are not functorially representable as structured sets and its morphisms as graphs.

Marquis observed that "[…] the conceptual quake set off by Freyd's result does not seem to have attracted the attention of philosophers of mathematics" and interpreted" […] this result as revealing the presence of a conceptual fault between the universe of homotopy types and the universe of (extensional) sets."

Di Liberti and Loregian generalized Freyd's theorem to the homotopical categories of a wide range of model categories. The aim of the talk is to discuss the interpretation offered by Marquis.

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"Informational equivalence" but "computational differences" of representations in mathematical practice

When solving a mathematical problem or reading a proof, drawing a well-chosen diagram may be very helpful. This well-known fact can be seen as an instance of a more general phenomenon. Using a diagram rather than sentences, reformulating a problem as an equation, choosing a particular notation rather than others: in all these cases, in a sense, we are only representing in a new form what we already knew; and yet, it can help us make progress. How is this possible?

Recent work on notations as well as on the use of diagrams in mathematics has put this question in focus, but there is little systematic philosophical work attempting to address it.

This paper focuses on a slogan, due to Herbert Simon (1978; 1987), which is regularly invoked in the literature to get to grips with this problem: two representations, writes Simon, may be "informationally" equivalent, but "computationally" different. Two representative recent uses of this slogan in philosophy of mathematics are Carter (2018, pp. 8 – 9), who compares "diagrammatic" and "formal" representations of simple graphs, and the work of Schlimm on the differences between different notations for numbers and (more recently) for propositional logic (see Schlimm and Neth 2008 and his work in progress, presented at previous APMP conferences).

The goal of this paper is to analyze Simon's idea and to discuss if and how it can be applied to representations in mathematics. Indeed, as is rarely appreciated, Simon's work originated in a specific cognitive science context and is based on an analogy with the computer science concept of data structures, which is tricky to apply to the external representations used in mathematics.

First, Simon's notion of "informational equivalence" requires the specification of precise translation procedures between two classes of representations (for instance, certain sorts of diagrams and certain lists of formulae). Such translation procedures, as I shall show, only make sense if we already have well-defined criteria for determining whether two representations (for instance, two diagrams drawn on paper) should count as "the same". Once this is clearly recognized, Simon's idea allows to give a clear meaning to some of the uses of the concept of "information" made in the literature on Euclidean diagrams (Miller 2007; Mumma 2008). Second, the notion of "computational" differences, based on the analogy with data structures in computer science, only makes sense in contexts where one can specify fully and in advance what sort of operations will be applied to representations, which, as I shall show, is difficult in most of the interesting mathematical cases.

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Essentially informal proofs about infinite time Turing machines

Larvor (2012) argued that a productive research program for philosophers of mathematical practice is to work toward a positive account of ‘essentially informal arguments’, or arguments “that would suffer some form of violence or essential loss” if they were recast as formal derivations (717). Philosophical work in this direction has often looked at proofs in specific mathematical sub-domains and arguing that some inferences within these proofs cannot be expressed in formal language, at least not with destroying the semantic meaning of these inferences (e.g., de Toffoli & Giardino, 2015). Other work has been to explain how inferences in visual representation systems that is not (and perhaps cannot be) expressed formally can nonetheless be valid, rigorous, or consensus-inducing (e.g., Manders, 2008).

We observe that most work of this type has occurred in mathematical sub-domains that rely heavily on visual reasoning and where diagrams are considered integral components of the proofs in these sub-domains. In the current paper, we extend this work to the domain of computability. The specific objects under investigation are Turing programs, which are formally defined as codes for programs to be carried out on Turing machines. However, in practice, Turing programs are usually represented in natural language as goal-directed descriptions of effective methods. By exploring the inferential structure of descriptions of goal-directed effective methods, we thus extend the work described in the first paragraph to a non-visual representation system employing natural language and to the domain of logic.

In particular, we analyze the inferential structure of goal-directed descriptions of effective methods in the context of infinite time Turing machines (Hamkins & Lewis, 2000), which for a variety of reasons provide particularly fertile ground for analysis. In Hamkins and Lewis’ paper, they define particular infinite time Turing programs using natural language terms such as recognize, search, find, guess, check, keep track, pay attention, and erase. The goal of the presentation is to illustrate that these terms play an ineliminable role in their proofs about infinite time Turing machines and we explain how their use is valid and rigorous. However, we also demonstrate that these terms are often undefined, context-dependent, and metaphorical. The conclusion will be that even in a logical domain such as computability, the use of informal representation systems is common in their proofs; the proofs are “essentially informal” in the sense that any translation of the proof into a formal derivation system would fundamentally change the semantic meaning of the proofs.

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Saturday, January 18 | 11:00 – 11:40 | Parallel session 1a | → HG E 33.3
Euclid’s Philosophical Commitments

My paper argues that Euclid’s Elements is committed to philosophical views about definition. The upshot of this is to gain an insight into Euclid’s methodological commitments pertaining to scientific definition, as well as to exemplify philosophical analysis of mathematical practise. Although Euclid was a mathematician, commentators have tried since antiquity to present Euclid as a philosopher. However, while scholars have engaged either in cosmological speculations about Euclid’s Elements (Proclus in Eucl., in Friedlein 1873; Hahn 2017) or in reconstructing the logical framework of Euclid’s mathematical proofs (Mueller 1981; Acerbi 2011; Acerbi forthcoming), my paper provides the first self-contained study of Euclid’s theory of definition.

While Euclid nowhere talks about his mathematical works, Euclid’s treatise nonetheless contains sufficient evidence for his implicit philosophical commitments and meta-mathematical background assumptions. Firstly, I provide a method for setting out philosophical presuppositions on the basis of (i) tacit assumptions, (ii) second-order language, and especially (iii) structural (methodical; linguistic) regularities in Euclid’s Elements. Secondly, by means of this method, I unveil aspects of Euclid’s logic, Euclid’s theory of science, and Euclid’s metaphysics. In particular, I argue that Euclid is committed to the following:

- a sharp distinction between species and differentiae; and
- priority in definition (A is prior in definition to B if A can be defined without B being defined, whereas B cannot be defined without A being defined).

For instance, Euclid rigidly defines the differentiae of mathematical species (such as 'even' and 'odd' for a number, and 'straight' for a line) in a way that is syntactically different from the way in which he defines the mathematical species (such as the number and the line) themselves. Therefore, we can infer that Euclid systematically distinguishes between species and differentiae. Moreover, since Euclid regularly introduces or defines terms prior in definition before defining terms posterior in definition, we can attribute the notion of 'priority in definition' to Euclid. In particular, Euclid shows himself committed to priority in definition of simpler over more complex items, of substances over their non-substantial attributes, of fundamental over derivative substances, of the whole over each of its parts, and of the genus over each of its species. Euclid shows himself committed to priority in definition, not only between different definienda, as well as between metaphysically different kinds of definiendum, but also between the definiendum and the terms in its definiens. The fact that Euclid tends to define a term prior to using it in the definiens of another term suggests that Euclid is also committed to the view that each of the terms in the definiens must be prior in definition to the definiendum.